Agribusiness Finance Report

Colorado State University

Extension

REAL OPTION ANALYSIS: AN OVERVIEW OF THE PROCESS AND HOW IT CAN BE APPLIED TO AGRIBUSINESS: PART III

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- *Calculating the asset value*

- *Calculating the option Value*

- *Those math formulas in laymen's terms*

In this last part of the Real Options (RO) Analysis series, we will describe how you would calculate actual option values such as those used in the application example from Part II.

Binomial discrete lattices, such as the one shown in Figure 1, depict the managerial flexibility inherent in RO analysis. In the figure, the option value depends on the value of the underlying asset (S) and the probability that the returns generated by the asset will increase (u) or decrease (d) in the next period (t_1). The same increase or decrease in the returns may occur in a subsequent period (t_2).

We will use some mathematical equations in this section that at first blush appear "nightmarish" in nature but you do need to be concerned with their derivations (although the readers are encouraged to ask us for more detail if they are interested). Rather we will show how these equations can be used by the reader in their individual calculations. For example, looking at equation 1:

$$(1) \quad \frac{\delta S}{S} = \mu(\delta t) + \sigma \varepsilon \sqrt{\delta t}$$

A percent change in the variable S (which is our asset) is a combination of a component that we assume never changes. We call this deterministic part, which is $\mu(\delta t)$. There is also a component that represents volatility or change, which we call stochastic. This is given by the following equation:

$$\sigma \varepsilon \sqrt{\delta t}.$$

μ, which is the growth piece of the deterministic component increases at a factor of time-steps, (δt), which

Figure 1

Today   **t_1**   **t_2**

u^2

u

S

ud **Binomial Lattice**

d

d^2
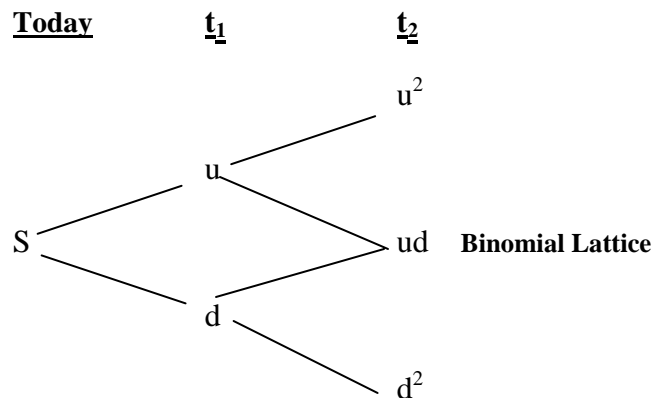
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can be thought of as periods—one day, one year, or whatever time period you choose to use. σ is the volatility parameter which also grows but at a rate of the square root of time. ε is a simulated variable that we won't need to use in our calculations. Although we keep the volatility remains constant over time, the *actual level* of uncertainty increases at a factor of

$$\sigma \sqrt{\delta t}.$$

Next we put equation 1 into a new form and we are almost ready to start our calculations:

$$(2) \quad \frac{\delta S}{S} = e^{\mu(\delta t) + \sigma \varepsilon \sqrt{\delta t}}$$

Equation 2 is used to calculate the probability of the stock's value increasing *(u)* and the probability of its value dressing *(d)* as follows:

$$(3) \quad u = e^{\sigma \sqrt{\delta t}} \quad \text{and} \quad d = e^{-\sigma \sqrt{\delta t}} = \frac{1}{u}$$

We can expand this growth by using a binomial tree, which which can be extended out for an infinite number of steps (for illustrative purposes, we have used only three time periods). Interestingly, if the volatility measure (σ) in equation 3 equals zero as is implicitly assumed in the discounted cash flow models, NPV and RO analyses are then equivalent and the lattice in Figure 1 would collapse into a straight line because all future cash flows are known with certainty. However, as we have already discussed in Parts I and II, uncertainty and changing volatility are common to most business ventures and the situation depicted in Figure 1 is the more realistic case—volatility could just as easily increase as decrease.

An example is useful in understanding how the binomial lattice is developed, and the first of two lattices is constructed using assumptions found in Table 1. The first lattice (found in Table 2) focuses on the value of the underlying asset, and how the asset value changes with increasing (decreasing) volatility in discrete time increments. To build Table 2's lattice, we start with the net present value of our stock (S) at time zero (S_0), multiply this value by the (*u*) and (*d*) factors using equation 3. Specifically,

$$u = e^{\sigma \sqrt{\delta t}} \text{ or } u = e^{0.10\sqrt{0.8}} = 1.0936 \text{ and } d = \frac{1}{u} = \frac{1}{1.0936} = 0.9144$$

$$100(1.0936) = 109.36$$
$$100(0.9144) = 91.44$$

The calculations are repeated to build the remainder of the lattice. Examining Table 2, the stock value has 6 potential values at the end of 5 time periods, and values range from 156.39 to 63.94. The ending values in period Step 5 depend on the initial value in Step 0 and the level of volatility (u and d) in each time step. It is the volatility that causes the up and down movements of the stock value—the higher the volatility, the higher the jumps up and down.

Just as a lattice may be built for the stock or asset value, a lattice may be built for the value of an option on that particular stock value. When building the option valuation lattice, we also need to calculate risk-neutral probabilities, *p*. This is not a probability in the sense that one will use it to make a prediction about future values; rather, it is simply an intermediate mathematical step, replacing the practice of discounting a risky set of cash flows at a risk-adjusted discount rate. By using a risk-free rate such as that of the readily available Treasury Bill or Note, we have eliminated some of the subjective nature of trying to determine an appropriate rate to discount uncertain cash flows. This

Table 1: The Assumptions for the Underlying Asset

Present Value of Stock	$100
Strike Price	$100
Expected Volatility	10.00%
Delta t (given)	0.8000
Discount Rate (dr)	5.00%
u	1.0936
d	0.9144

Table 2: The Underlying Lattice

Step 0	Step 1	Step 2	Step 3	Step 4	Step 5
100	109.36	119.56	130.78	143.01	156.39
	91.44	100.00	109.36	119.59	130.78
		83.62	91.44	100.00	109.36
			76.47	83.62	91.44
				69.92	76.47
					63.94

Source: Real Options Toolkit

means that "The risk adjustment is performed on the probabilities of future option cash flows," rather than on the cash flows themselves. (Mun 2002). The risk neutral probabilities are calculated as:

$$(4)\ \$1 = \left[(p)u + (1-p)d\right]e^{-dr(t)}$$
$$e^{-dr(t)} = pu + d - pd$$
$$e^{-dr(t)} = p(u-d) + d$$
$$p = \frac{e^{-dr(t)} - d}{u - d}$$

Using Equation 4 and the assumptions from Table 1, we can calculate p and $1\text{-}p$. The initial assumptions as well as p and $1\text{-}p$ are now included in Table 3.

$$p = \frac{e^{-dr(t)} - d}{u - d} = p = \frac{e^{-.05(0.8)} - 0.9144}{1.0936 - 0.9144} = 0.7055 \text{ and}$$
$$1 - p = 1 - 0.7055 = 0.2945$$

Table 3. Values for p and $1\text{-}p$

Variables	
Present Value of Stock	$100
Strike Price	$100
Expected Volatility	10.00%
Delta t (given)	0.8000
Discount Rate (dr)	5.00%
u	1.0936
d	0.9144
p	0.7055
1-p	0.2945

Now we are ready to calculate the values for the *option lattice* (Table 4). In order to do this, we will work backwards starting at the last step, which in this example would be Step 5.

Starting with $156.39 (Table 2, Step 5), we know that in this case, the call option has value; we can buy the

stock at $100 and sell it for $156.39 making the option worth $56.39.[8] However, if the price of the stock were $91.44 (Table 2, Step 5), then we would not exercise the option because to do so would result in a loss—buying at $100 and selling at $91.44. In this case, we would let the option expire; its value equals 0. To calculate the rest of the option values in Table 4, we continue to work our way backwards by discounting the weighted average of potential future option values using the risk-neutral probabilities:
which is the value in Step 4.

$$(5)\left[p(u) + (1 = p)(d)\right]e^{(-rf*dt)}\text{or using our example}$$

$$\left[0.7055(56.39) + (0.2945)(30.78)\right]e^{(-0.04)} = 46.93$$

In other words, the value associated with keeping this option open would be $46.93. We continue this backwards induction until we eventually get to step 0 and the actual value of keeping this option open as of today. In this case, the $19.59 represents the value associated with keeping this option open in order to potentially earn higher returns in the future. Each value in the table represents the "discounted weighted average of potential future option values using the risk-neutral probability" (Mun 2002). The binomial lattice examples in Tables 2 and 4 illustrate the mechanics behind RO analysis.

Table 4. The Option Valuation Lattice:

Step 0	Step 1	Step 2	Step 3	Step 4	Step 5
Today					
19.59	24.82	31.09	38.47	46.93	56.39
	9.80	13.23	17.73	23.51	30.78
		2.91	4.30	6.34	9.36
		0.00	0.00	0.00	0.00
				0.00	0.00
					0.00

Each step represents a time period, which can be any framework chosen by the user. Here we can assume these steps represents years.

Source: Real Options Toolkit

References:

Mun, J. (2002) *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. John Wiley & Sons, Inc. New Jersey

Real Options Toolkit. (2002) J. Mun as part of *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions*. John Wiley & Sons, Inc. New Jersey.