

CHAPTER 9

INTEREST RATE FUTURES

INTRODUCTION

Without a doubt, interest rate futures contracts, which are contracts for debt instruments, are the most successful type of futures contracts. A quick review of the futures quote page in the *Wall Street Journal* will show this. The volume of trade in futures contracts on debt instruments is almost larger than that for all other contracts combined. The 1970s was the decade of physical commodities, currencies emerged with prominence in the early 1980s, stock and other index contracts have made strong gains in volume in the late 1980s and 1990s, but debt instruments were the contracts of the 1980s and 1990s. The volume of trade in these futures instruments continues to grow.

Trade in interest rate futures contracts began during October 1975 when the Chicago Board of Trade (CBOT) introduced the Government National Mortgage Association (GNMA or "Ginnie Mae") contracts. Since this time, exchanges have introduced a large number of interest-rate-based contracts. Many of the contracts have been failures, attracting very little volume and trading activity. However, several contracts have been huge successes.

The growth in volume of trade in interest rate futures contracts was due to the increased risk created by the interest rate volatility beginning in the late 1970s, which then carried through to the 1980s, and resurfaced to an extent in the mid-1990s. Prior to the 1970s, inflation and interest rates were stable and predictable. The late 1970s witnessed double-digit inflation rates. Increases in interest rates were implemented by the U.S. Federal Reserve in the early to mid-1980s to slow the economy and reduce inflation. The late 1980s and 1990s witnessed periodic adjustments to interest rates to both slow and stimulate the economy as inflation was brought down to the current relatively low levels. Revenue and cost uncertainty for lenders and borrowers following these substantial changes in interest rates have made hedging essential. The large movements in interest rates have also attracted speculative capital which has helped this market grow.

The 1980s and 1990s also saw an industrialization and integration process in the

markets for capital and debt. Markets for debt are now always national and usually global. This process has been facilitated by standardization practices in capital markets and improved communication systems. Individual debts and debt instruments are standardized, pooled, and traded. Borrowers receive needed capital, lenders receive an income stream for a defined time period with known default risk, and financial institutions serve large roles as intermediaries—coordinating the interaction of those wishing to borrow and those wishing to loan capital. Financial institutions standardize and pool mortgages, bonds and commercial paper, and sell the resulting instruments to investors and businesses seeking interest returns on funds available to loan. Financial institutions purchase government and corporate bonds, bills, notes and other debt instruments and sell these to customers desiring income streams.

Lenders of capital face the risk of having the value of the debt instruments that they hold decrease. If a firm that operates a mutual fund purchases a debt instrument, for example a government bond, that yields 5.5 percent per annum and after two years the interest rate increases to 6 percent, there is an opportunity cost of lost income. Likewise, borrowers of capital are at risk of having the value of debt instruments they are under obligation to repay increase. If a food processing business issues a debt instrument, such as commercial paper, that yields 6.75 percent per annum after 180 days and the interest rate decreases to 6.5 percent before funds are returned, then there is an opportunity cost of paying higher than market rates. Further, the financial intermediaries that coordinate the flow of capital and debt instruments between lenders and borrowers are subject to substantial risk. This risk creates demand for vehicles to hedge the many types of debt instruments. This risk also attracts speculators.

All of the significant trade in U.S. interest rate futures occurs at the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT). The two exchanges have essentially specialized in the trade of different products. The IMM trades debt instruments with very short maturities. Eurodollar deposits and Treasury Bills are the most popular contracts. By contrast, the CBOT trades debt instruments with long-term maturities. These instruments include treasury bonds and treasury notes. The CBOT also offers a contract on 30-day Federal Funds, which are a short-term debt instrument, and a longer-term Municipal Bond Index. However, the volume of trade in these products is small.

Eurodollars are debt instruments for 90-day dollar deposits held in a European bank. The interest paid on this debt instrument is the London Interbank Offered Rate (LIBOR). Treasury bills are short-term U.S. government securities with typical maturities of 90 days and one year. The 90-day T-bill is a very active market. T-bonds and T-notes are U.S. government securities that make biannual interest payments to the holder. T-bond futures contracts are for bonds that mature in no less than 15 years. Separate futures contracts are offered on T-notes which mature in 10, 5, and 2 years. The 10-year T-note is the most active contract.

The remainder of this chapter will focus on the most active futures contracts. The short-term contracts considered are Eurodollars and treasury bills. The long-term contracts discussed are treasury bonds and treasury notes. Eurodollars and T-bills are priced in the futures market in a very similar fashion. The concepts that apply to one, apply to the other. The same is true for T-bonds and T-notes. The contracts are priced in a very similar fashion, and the general concepts that apply to one apply to the other. The main differences between the contracts are in their usefulness in hedging debt of various lengths of maturity. Our discussion will focus on this aspect of the futures contracts and trading. Table 9.1 presents the specifications of these interest rate contracts.

Much of the complexity in the different interest rate contracts emerges in the

Instrument	Exchange	Par Value	Maturity	Quote	Basis Point Value
Eurodollars	CME	\$1 million	90 days	100-yield	\$25
Treasury bills	CME	\$1 million	13 weeks	100-yield	\$25
Treasury notes	CBOT	\$200,000	2 years	points & 32nds	\$62.50
Treasury notes	CBOT	\$100,000	5 years	points & 32nds	\$31.25
Treasury notes	CBOT	\$100,000	10 years	points & 32nds	\$31.25
Treasury bonds	CBOT	\$100,000	15 years	points & 32nds	\$31.25

TABLE 9.1
Debt Instrument Futures
Contract Specifications

delivery process. Significant delivery of the actual debt instruments takes place and the contracts are designed so that a multitude of different instruments within each category can be delivered. For example, the variety of treasury bonds that can be delivered on the CBOT T-bond contract is quite large. And much of the complexity in the contract emerges in determining this delivery process and valuing the variety of instruments that can be delivered on a contract that has a fixed specification. This chapter will not address this issue. Like other futures markets, delivery on interest rate futures contracts is the realm of professional traders who concentrate on providing this service. The focus of this chapter is on the mechanics of trading each contract and on use of the contracts in a hedging program.

INTEREST RATE AND DEBT PRICING BASICS

This section discusses what the different debt instruments are, how these debt instruments are priced, and how interest rate futures contracts for each instrument are priced. The instruments and pricing procedures are rather common to the various markets for debt instruments. But the procedures are rather different from contracts for physical commodities, exchange rates, and indexes.

We will begin with the contracts on debt instruments with short-term maturities. The two contracts of interest are for Eurodollars and treasury bills. Debt instruments with short-term maturities usually call for the payment of a fixed amount of cash at maturity. This is the *par value* of the instrument. For example, a T-bill may pay \$1 million to the holder after 90 days. The amount of money that the holder must pay for the debt instrument, the *purchase price* or *value*, varies with the interest rate. The more the purchaser pays, the lower the interest rate of the instrument. Again, we are talking about actual T-bills and not the T-bill futures contract.

The interest rate for T-bills is quoted in terms of a *discount yield*. The discount yield is the interest rate associated with the T-bill. T-bills have a fixed par value at maturity, and the price is determined by the par value less the discount implied by the discount yield. The price is determined as follows:

$$\begin{aligned}
 \text{Price} &= \text{Par Value at Maturity} - \text{Discount Yield} \times \text{Par Value} \times \frac{\text{Days-to-Maturity}}{\text{Days per Year}} \\
 &= \text{Par Value} - \text{Discount} \times \frac{\text{Days-to-Maturity}}{\text{Days per Year}} .
 \end{aligned}$$

The discount yield equals the par value less the price, or the discount, as a percent of the par value converted to an annual interest rate

$$\begin{aligned}\text{Discount Yield} &= \frac{\text{Par Value} - \text{Price}}{\text{Par Value}} \times \frac{\text{Days-per-Year}}{\text{Days-to-Maturity}} \\ &= \frac{\text{Discount}}{\text{Par Value}} \times \frac{\text{Days-per-Year}}{\text{Days-to-Maturity}}.\end{aligned}$$

For a T-bill with a \$1 million par value, 90 days to maturity, and a discount yield of 6 percent, the price is

$$\begin{aligned}\text{Price} &= \$1,000,000 - 0.06 \times \$1,000,000 \times \frac{90}{360} \\ &= \$1,000,000 - \$15,000 = \$985,000.\end{aligned}$$

Or if the discount yield can be recovered for the same T-bill with the price of \$985,000,

$$\begin{aligned}\text{Discount Yield} &= \frac{\$1,000,000 - \$985,000}{\$1,000,000} \times \frac{360}{90} \\ &= \frac{\$15,000}{\$1,000,000} \times \frac{360}{90} = 0.06 = 6\%.\end{aligned}$$

Like T-bills, Eurodollar deposits also have a fixed par value at maturity, a discount yield, a discount, and a price. However, the interest rate is quoted in terms of an *add-on yield*. The add-on yield is the ratio of the discount and the price of the instrument multiplied by a second ratio which converts the first to an annual interest rate. The add-on yield is

$$\begin{aligned}\text{Add-On Yield} &= \frac{\text{Discount}}{\text{Par Value} - \text{Discount}} \times \frac{\text{Days-per-Year}}{\text{Days-to-Maturity}} \\ &= \frac{\text{Discount}}{\text{Price}} \times \frac{\text{Days-per-Year}}{\text{Days-to-Maturity}}.\end{aligned}$$

So, for Eurodollar deposits with a \$1 million face value, 90 days from maturity, and a discount yield of 6 percent, the add-on yield is

$$\begin{aligned}\text{Add-on Yield} &= \frac{\$15,000}{\$1,000,000 - \$15,000} \times \frac{360}{90} \\ &= \frac{\$15,000}{\$985,000} \times \frac{360}{90} = 0.0609 = 6.09\%\end{aligned}$$

Notice, the T-bill and Eurodollar deposits both have the same discount yield, but the reported interest rate is higher for the Eurodollar than for the T-bill because of the reporting convention.

Futures price quotes for the T-bill and Eurodollar contracts do not report the yields directly but use the IMM Index. The index is 100 minus the interest rate associated with the instrument or

$$\text{IMM Index} = \text{Futures Price} = 100.00 - \text{Interest Rate.}$$

IMM Index for the T-bill futures contract uses the discount yield

$$\text{IMM Index} = \text{Futures Price} = 100.00 - \text{Discount Yield}$$

so that if the yield is 6 percent, the futures price is

$$\text{IMM Index} = 100.00 - 6.0 = 93.00.$$

The price of the futures contract is quoted as *93.00 points* or *9,300 basis points*. The interest rate is in percentage terms, 6 percent, and the price of the instrument is 93.00 percent of par. Basis points are 1/100th of 1 percent.

IMM Index for the Eurodollar futures contract uses the add-on yield

$$\text{IMM Index} = \text{Futures Price} = 100.00 - \text{Add-On Yield}$$

so when the yield is 6.09 percent, the futures price is

$$\text{IMM Index} = 100.00 - 6.09 = 92.91.$$

The price of the futures contract is 92.91 points or 9,291 basis points. The price of the instrument is 92.91 percent of par. In these two examples, the reported interest rate differs by nine basis points.

Table 9.2 presents futures price quotes for these two contracts reported in the *Wall Street Journal* for October 2, 1997. Notice the extent of the maturities traded for Eurodollars and notice the volume and open interest of both contracts.

An interesting feature of this pricing method is that the IMM Index decreases with increases in the interest rate. For example, the price quoted for nearby T-bills in Table 9.2 is 95.03 points. Suppose the next day the price decreases to 94.90 points. This implies the interest rate has risen from 4.97 percent to 5.10 percent. The implication is that beginners need to be careful working through trading examples. You are not trading the interest rate directly. Rather, the IMM Index is pricing the contract as a percent of the par value of the contract size. This is roughly the same convention with which the underlying debt instrument is traded.

Long-term debt instruments are very different from their short-term counterparts, are priced rather differently, and the pricing is more complex. Again, we are referring to the debt instrument and not the futures contract. Short-term debt instruments are almost always *pure discount bonds*. The investor pays a price for the instrument that is some discount to the par value and is returned the par value at maturity. For example, a firm may buy a T-bill with a par value of \$10,000 after 90 days for \$9,875. After 90 days the T-bill is redeemed for \$10,000 and the investor has earned 5 percent per annum. Long-term debt instruments are rarely pure discount bonds. Investors usually receive period interest payments termed *coupons*. For example, a mutual fund may purchase a treasury bond with a \$100,000 par value, 15-year maturity, and an 8 percent per annum biannual interest payment. The bond entitles the firm to receive

TABLE 9.2 Futures Prices for Eurodollar and T-Bill Futures, October 2, 1997

	Open	High	Low	Settle	Change	Yield Settle	Yield Change	Open Interest
EURODOLLAR (CME) \$1 million; pts of 100%								
Oct	94.25	94.25	94.24	94.25	—	5.75	—	24,890
Nov	94.22	94.22	94.22	94.22	+01	5.78	-.01	12,262
Dec	94.19	94.21	94.19	94.20	+01	5.80	-.01	589,353
Mr98	94.15	94.18	94.14	94.16	+02	5.84	-.02	424,920
June	94.07	94.11	94.06	94.09	+03	5.91	-.03	315,650
Sept	93.99	94.04	93.98	94.01	+03	5.99	-.03	242,998
Dec	93.88	93.92	93.86	93.90	+03	6.10	-.03	223,182
Mr99	93.87	93.91	93.85	93.88	+03	6.12	-.03	150,575
June	93.82	93.86	93.81	93.84	+03	6.16	-.03	119,948
Sept	93.79	93.83	93.78	93.81	+03	6.19	-.03	98,821
Dec	93.72	93.76	93.71	93.74	+03	6.26	-.03	86,455
Mr00	93.72	93.76	93.72	93.74	+03	6.26	-.03	71,511
June	93.69	93.73	93.68	93.71	+03	6.29	-.03	59,705
Sept	93.66	93.70	93.65	93.68	+03	6.32	-.03	50,295
Dec	93.59	93.64	93.59	93.62	+03	6.38	-.03	40,627
Mr01	93.59	93.64	93.59	93.62	+03	6.38	-.03	35,800
June	93.56	93.61	93.56	93.59	+03	6.41	-.03	31,675
Sept	93.53	93.57	93.53	93.56	+03	6.44	-.03	29,616
Dec	93.46	93.52	93.46	93.50	+04	6.50	-.04	17,451
Mr02	93.46	93.52	93.46	93.50	+04	6.50	-.04	16,673
June	93.43	93.49	93.43	93.47	+04	6.53	-.04	13,091
Sept	93.40	93.46	93.40	93.44	+04	6.56	-.04	11,286
Dec	93.34	93.40	93.34	93.37	+03	6.63	-.03	7,678
Mr03	93.34	93.40	93.34	93.37	+03	6.63	-.03	6,432
June	93.30	93.36	93.30	93.33	+03	6.67	-.03	4,853
Sept	93.32	93.33	93.31	93.30	+03	6.70	-.03	4,290
Dec	93.22	93.26	93.22	93.24	+03	6.76	-.03	5,206
Mr04	93.22	93.26	93.22	93.24	+03	6.76	-.03	4,436
June	93.19	93.23	93.19	93.21	+03	6.79	-.03	6,251
Sept	93.16	93.20	93.16	93.18	+03	6.82	-.03	3,955
Dec	93.10	93.14	93.10	93.12	+03	6.88	-.03	4,360
Mr05	93.10	93.14	93.10	93.12	+03	6.88	-.03	2,005
June	93.05	93.10	93.05	93.08	+03	6.92	-.03	2,522
Sept	93.03	93.07	93.03	93.05	+03	6.95	-.03	2,068
Dec	92.97	92.99	92.97	92.99	+03	7.01	-.03	1,730
Mr06	92.97	92.99	92.97	92.99	+03	7.01	-.03	2,799
June	92.93	92.95	92.93	92.95	+03	7.05	-.03	1,819
Sept	92.89	92.91	92.89	92.91	+03	7.09	-.03	1,750
Dec	—	—	—	92.85	+03	7.15	-.03	1,485
Mr07	—	—	—	92.85	+03	7.15	-.03	1,271
Est vol 355,105; vol Wed 449,623; open int 2,733,875, +23,-111								
TREASURY BILLS (CME) \$1 million; pts. of 100%								
Dec	95.03	95.04	95.03	95.03	+02	4.97	-.02	5,031
Est vol 1,187; vol Wed 688; open int 8,645, +270								

Source: Wall Street Journal.

coupon payments worth \$4,000 twice per year (\$8,000 = \$100,000 × 8% coupon) for 15 years, and then the bond is redeemed for the par value.

How much should the firm pay for this bond? The price of the bond is determined with the following formula which calculates the present value of a stream of interest payments with a lump sum end payment. The bond price is

$$Price = \left[\sum_{t=1}^{2n} \frac{I_t}{(1 + \frac{1}{2} i)^t} \right] + \frac{Par\ Value}{(1 + \frac{1}{2} i)^{2n}}$$

where I_t is the biannual coupon payment, i is the annual market interest rate, and the bond matures in n years. The summation is the present value of the interest payments. Notice the biannual payments. The second part of the right-hand side is the discounted par value. It is important to notice that there are two interest rates in the valuation. The bond pays a coupon which is the interest rate of the bond. However, the price of the bond, or what the bond is worth, depends on the market interest rate. The following examples will make this clear.

Suppose a bond has the par value of \$100,000, matures in 15 years, pays an 8 percent per annum coupon, and the current market interest rate is 8 percent. The price of the bond is

$$\begin{aligned} Price &= \left[\sum_{t=1}^{2 \times 15} \frac{4000_t}{(1 + \frac{1}{2} 0.08)^t} \right] + \frac{\$100,000}{(1 + \frac{1}{2} 0.08)^{2 \times 15}} \\ &= \$69,168 + \$30,832 = \$100,000. \end{aligned}$$

The price of the bond is equal to the par value because the interest paid by the bond is equal to the market interest rate. This market interest rate is the opportunity cost of holding the bond.

Now, suppose that the market interest rate is 6.5 percent. The price of the bond with an 8 percent per annum coupon is

$$Price = \left[\sum_{t=1}^{2 \times 15} \frac{4000_t}{(1 + \frac{1}{2} 0.065)^t} \right] + \frac{\$100,000}{(1 + \frac{1}{2} 0.065)^{2 \times 15}} = \$114,236.$$

The price of the bond is at a premium to the par value because the bond returns 8 percent per annum but the market rate is 6.5 percent. The convention used in the bond market is that the price of this bond would be quoted as *114.24 percent of par*.

Suppose that the market interest rate is 9.25 percent. The price of the bond with an 8 percent per annum coupon is

$$Price = \left[\sum_{t=1}^{2 \times 15} \frac{4000_t}{(1 + \frac{1}{2} 0.0925)^t} \right] + \frac{\$100,000}{(1 + \frac{1}{2} 0.0925)^{2 \times 15}} = \$89,967.$$

The price of the bond is at a discount to the par because the bond returns 8 percent per annum but the market rate is 9.25 percent. The convention used in the bond market is that the price would be quoted as *89.97 percent of par*.

A further reporting convention used in markets for long-term government securities is that instead of using decimals in the percent of par quotations, the market uses *points and 32nds of par*. For example, a treasury bond may be priced as 114-08. This implies

$$114-08 \text{ points and } 32\text{nds} = 114 \text{ and } 08/32\text{s points} = 114.25 \text{ percent of par.}$$

Likewise, if the T-bond is priced as 89-31, then

$$89-31 \text{ points and } 32\text{nds} = 89 \text{ and } 31/32\text{s points} = 89.97 \text{ percent of par.}$$

It is not possible to take a quotation in points and 32nds of par, where par is for an 8 percent coupon bond, and use a simple formula to convert this to an interest rate quotation. Instead, you would have to solve the formula for the market interest rate as a function of the price, par value, and using the 8 percent coupon payments. This is not possible.

But, we can do one of two things to calculate the yield, or interest rate, implied by a bond price quotation. First, we can use trial-and-error with the bond price formula. We plug different interest rates into the formula and see which one gives us a price closest to the quotation. This is very easy to do with a spreadsheet. We can program the bond price formula into a spreadsheet and rapidly examine prices for different interest rates. Second, we can use the following formula which approximates the yield:

$$\text{Yield} = \frac{\text{Coupon per Annum} + \frac{(\text{Par} - \text{Price})}{\text{Years}}}{\frac{(\text{Par} + \text{Price})}{2}}$$

Working with the numbers of the first example, we see that the approximation gives

$$\text{Yield} = \frac{\$8,000 + \frac{(\$100,000 - \$114,236)}{15}}{\frac{(\$100,000 + \$114,236)}{2}} = 0.0658 = 6.6\%$$

which is just slightly different from the true yield of 6.5 percent.

The long-term debt instruments traded at the CBOT are reported in points and 32nds of par. The contracts are specified for T-bonds and T-notes which pay 8 percent per annum biannual coupons. Deliverable T-bonds should mature in not less than 15 years. Contracts are offered on T-notes that, at the expiration of the contract, have maturities of 10, 5, and 2 years. Table 9.3 presents price quotes for T-bond and T-note futures contracts reported in the *Wall Street Journal* for October 2, 1997. Again, notice the volume of trade and open interest.

Like, the short-term contracts, pricing the futures contracts in points and 32nds of par results in the price of the contract decreasing with increases in the interest rate. Again, if you are not familiar with bond pricing, you need to be careful working through trading examples. You are not trading the interest rate directly. Rather, you are trading the contract as a percent of the par value. This is what is done in the underlying T-bond and T-note markets.

TABLE 9.3 Futures Prices for T-Bonds and T-Notes, October 2, 1997

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
TREASURY BONDS (CBOT) \$100,000; pts. 32nds of 100%								
Dec	116-10	116-28	116-06	116-20	+9	118-08	100-08	644,599
Mr98	116-03	117-24	115-21	116-10	+9	117-24	104-21	38,468
June	115-31	116-06	115-31	115-31	+10	116-14	104-03	4,238
Sept	—	—	—	115-21	+9	115-00	103-22	1,957
Dec	—	—	—	115-12	+9	114-18	103-13	4,669
Est vol 420,000; vol Wed 618,284; open int 693,986, +36,496								
TREASURY NOTES (CBOT) \$100,000; pts. 32nds of 100%								
Dec	110-20	110-30	110-18	110-25	+5	110-30	104-10	377,787
Mr98	110-09	110-18	110-09	110-14	+5	110-18	105-24	14,729
Est vol 82,183; vol Wed 110,086; open int 392,519, +9,683								
5-YR TREASURY NOTES (CBOT) \$100,000; pts. 32nds of 100%								
Dec	107-24	107-30	107-22	107-26	+2.5	107-30	104-005	234,928
Est vol 36,615; vol Wed 55,187; open int 235,001, +6,195								
2-YR TREASURY NOTES (CBOT) \$200,000; pts. 32nds of 100%								
Dec	103-21	103-227	103-205	103-22	+1.0	103-227	102-265	43,212
Est vol 2,000; vol Wed 3,108; open int 43,212, +777								

Source: *Wall Street Journal*.

The volatile interest rates of the high-inflation years in the late 1970s and early 1980s gave huge impetus to interest rate futures. All of a sudden, business firms faced very significant risks they had not faced before. When risk of this magnitude emerges, there is great demand for a way to manage exposure to that risk. This is the role of futures trade.

INTEREST RATE FUTURES TRADING EXAMPLES

Bonding pricing is not all that difficult, but it is rather different from that of other assets. However, trading futures contracts on debt instruments is relatively straightforward once you understand the basics of bond pricing. We will work a few trading examples. These trading examples could be speculative or could be part of a hedge.

Suppose that in October a trader sells one December T-bill contract priced at 95.00 points. In December, the contract is trading at 94.50 points. The trader offsets the position by buying the contract. The net change in the value of the position is the price of the instrument at the time of sale less the price of the instrument at the time of purchase, which is

$$\begin{aligned}
& (95.00 \text{ points} \times \$1 \text{ million}) - (94.50 \text{ points} \times \$1 \text{ million}) \times (90 \text{ days} / 360 \text{ days}) = \\
& (95.00\% \times \$1 \text{ million}) - (94.50\% \times \$1 \text{ million}) \times (90 \text{ days} / 360 \text{ days}) = \\
& (\$950,000 - \$945,000) \times (90 \text{ days} / 360 \text{ days}) = \\
& (\$5,000) \times (90 \text{ days} / 360 \text{ days}) = \$1,250.
\end{aligned}$$

The net change in the value of the position can also be calculated using the price change.

$$\begin{aligned}
& (95.00 - 94.50) \text{ points} \times \$1 \text{ million} \times (90 \text{ days} / 360 \text{ days}) = \\
& 50 \text{ basis points} \times \$1 \text{ million} \times (90 \text{ days} / 360 \text{ days}) = \\
& 0.50\% \times \$1 \text{ million} \times (90 \text{ days} / 360 \text{ days}) = \\
& (\$5,000) \times (90 \text{ days} / 360 \text{ days}) = \$1,250
\end{aligned}$$

The discount yield increased from 5 percent to 5.5 percent between October and December, and the trader made money by selling the T-bill contract. The procedure for calculating the gain or losses from trading the Eurodollar contract are exactly the same as for the T-bill.

Rather than performing the preceding calculations, it is easier to keep track of the change in the number of basis points associated with a position—the sell price minus the buy price—and then multiply this change by the value of a one-basis-point change in the contract. A one-basis-point or a 0.01 percent change in a contract with a \$1 million par value would be \$100, but we need to remember that the debt instrument matures in 90 days.

$$\begin{aligned}
& \$1 \text{ million} \times (90 \text{ days} / 360 \text{ days}) \times 1 \text{ basis point} = \\
& \$1 \text{ million} \times (90/360) \times 0.0001 = \$25
\end{aligned}$$

Thus, the value of a one-point change in the T-bill or Eurodollar contract is \$25.

As a second example, suppose that in October a trader sells eight March Eurodollar contracts at 94.14 points. The next day, the contract is trading at 94.16 points, and the trader offsets this position by buying eight contracts. The net change in the value of the position is

$$\begin{aligned}
& (94.14 - 94.16) \text{ points} \times (\$1 \text{ million} \times (90 \text{ days} / 360 \text{ days})) \text{ per contract} \times \\
& 8 \text{ contracts} = \\
& -2 \text{ basis points} \times \$25 \text{ per point per contract} \times 8 \text{ contracts} = \\
& -\$50 \text{ per contract} \times 8 \text{ contracts} = -\$400.
\end{aligned}$$

The yield decreased from 5.86 percent to 5.84 percent between the two days, and the trader lost money by selling the Eurodollar contracts.

Since T-bonds and T-notes are quoted in points and 32nds, one basis point is 1/32nd of a point. The change in the value of a contract following a one-basis-point change in the quote is

$$\text{Par Value} \times 1/32 \text{ of 1 point} = \text{Par Value} \times 0.03125\%.$$

The par value of a T-bond, T-note (10-year), and T-note (5-year) is \$100,000, so a one-basis-point change is worth \$31.25 per contract. The par value of a two-year T-note contract is \$200,000, so the value of a one-basis-point change is double. It is easier to work with T-bond and T-note prices if they are converted to decimals. In decimals, a

one-full-point change is worth \$1,000 (32 32nds per full point \times \$31.25), and in this case a one-basis point change is worth \$10. Be careful keeping track of the units.

Suppose that in October a trader sells one December T-bond contract at 116-16, which is a yield of 6.285 percent. In December, the trader offsets the contract at 115-08, a yield of 6.403 percent. The net change in the value of the position is

$$\begin{aligned} & (116-16 - 115-08) \text{ points and 32nds} \times \$31.25 \text{ per 32nd point per contract} = \\ & (1-08) \text{ points and 32nds} \times \$31.25 \text{ per 32nd point per contract} = \\ & (32+8) \text{ 32nds} \times \$31.25 \text{ per 32nd point per contract} = \\ & (40) \text{ 32nds} \times \$31.25 \text{ per 32nd point per contract} = \$1,250 \end{aligned}$$

or converting the 32nds to decimals,

$$\begin{aligned} & (116.5 - 115.25) \text{ points} \times \$1,000 \text{ per point per contract} = \\ & (1.25) \text{ points} \times \$1,000 \text{ per point per contract} = \$1,250. \end{aligned}$$

The yield increased from 6.285 percent to 6.403 percent between October and December, and the trader made money by selling the T-bond contract.

The important thing to remember with interest rate futures contracts is that you are trading the discount to par. If the price that represents the discount to par increases, the discount is decreasing, and the interest rate is decreasing. This point is worth repeating, especially since much of this section has dealt with the details of debt instrument pricing. Do not lose the larger perspective because of these details. The price you are trading for the debt instrument underlying the futures contract is some measure of the discounted market value of the instrument. As interest rates rise, the discounted market value and the futures price will fall. Likewise, as interest rates fall, the discounted market value and the futures prices will rise.

INTEREST RATE HEDGING EXAMPLES

This section presents several example interest rate hedges. As with all hedging, the trader has a position in the cash market or will have a position in the cash market that will change in value if the price of the underlying asset for a debt instrument changes. In this case, the assets are loaned or borrowed money and capital. The trader has purchased or will purchase a debt instrument, or is or will be under obligation to repay a debt instrument.

A person or business who has borrowed or will loan is long spot market interest rates. This trader has sold or will purchase a debt instrument. If interest rates increase, the trader benefits. For example, if a business borrows money at a fixed interest rate and the market interest rate increases, the firm benefits. Likewise, if a trader promises to deliver a T-Bill in the future and interest rates increase, the trader will have an unanticipated decrease in the cost of meeting that obligation. A person or business who has loaned or will borrow is short spot market interest rates. This trader has purchased or will sell a debt instrument. If interest rates decrease, the trader benefits. For example, if a business loans money at a fixed interest rate and the market interest rate decreases, the firm benefits. Likewise, the trader holds a T-Bond and interest rates decrease, the trader can sell the T-Bond for more than the purchase price. An important component of interest rate hedging involves recognizing whether the spot position is long or short—does the trader stand to gain or lose from an increase or decrease in interest rates? This must be determined first.

A second component involves matching the cash instrument with an appropriate futures contracts. What is the maturity of the debt instrument? Long-term, intermediate-term, and short-term interest rates can be rather different and can respond to different market events. Traders may be interested in buying or selling debt instruments with maturities from the very short-term, such as 30 days, to the very long-term, 25 years, for example. And as we have seen, there are futures contracts for short-term debt, T-bill and Eurodollar futures contracts, intermediate term debt, two-year and five-year T-notes futures contracts, and long-term debt, T-bond futures contracts.

It is also important to recognize that the maturity of the debt instrument and the length of the hedge are two very different time components of the hedging problem. For example, suppose a trader knows that in three months she will have \$1 million with which to purchase 90-day T-bills. The trader will purchase 90-day T-bills in three months and that rate may or may not be the same as the rate for 180-day T-bills today. The market consensus of the rate that the trader will receive is the current price of T-bill futures that expire in three months. The maturity of the debt instrument and the length of the hedge may be the same. For example, suppose a business with a one-year loan with a floating interest is interested in hedging or locking in a fixed rate. The firm can take an opposite position in the futures market of a contract that the loan rate follows. Then, the length of the hedge is equal to the length of the loan. But the T-bill example earlier shows that the two time periods are different things. The point we are making is that there are many time periods and time horizons to consider with debt instruments. You need to keep the maturity of the debt instrument and the length of the hedge separate in your mind.

Let's work a couple of basic hedges. The first hedge is for a portfolio manager who will purchase T-bills in the future. It is March 15 and the manager learns that he will have \$985,000 on June 15 with which to purchase 90-day T-bills. The June contract is trading for 93.75 points, or a discount yield of 6.25 percent. The future price implies the manager can lock in the price of one T-bill with a par value of \$1 million for \$984,375. The manager takes a long position and buys one contract. The complete hedge is summarized in Table 9.4.

Suppose that on June 15, the June T-bill contract is trading at 95.25 points and the yield on T-bills is 4.75 percent. The yield implied by the futures price is also 4.75 percent. The manager sells one June T-bill contract to offset the long position. The net on the futures trade is +150 basis points and the return is \$3,750. The manager pays \$988,125 for a 90-day T-bill with a par value of \$1 million and uses the gains from the futures market to reduce this cost to \$984,375. The annualized yield on the T-bill is therefore 6.25 percent. The portfolio manager used the futures market to lock in the T-bill yield.

TABLE 9.4

Hedging the Future
Purchase of Treasury
Bills by a Portfolio
Manager

Date	Cash Market	Futures Market
March	Anticipate purchasing T-bill with \$1 million par value in June for \$984,375. Expected yield of 6.25%.	Buy one June T-bill contract at 93.75 points. Implied yield is 6.25%.
June	Purchase T-Bill with \$1 million par value for \$988,125. Loss of \$3,750.	Sell 1 June T-Bill contract at 95.25 points. Gain of \$3,750.
	Net change of \$0. Actual T-Bill yield of 6.25%.	

The second hedge is for a securities trader that has agreed to sell T-bills in the future. On September 15, a securities trader contracts to deliver one 90-day T-bill with a par value of \$1 million on December 15 to a customer for \$984,375. This price allows the trader to make a small profit. The trader has contracted to deliver a T-bill with a 6.25 percent yield. The December futures contract is trading for 93.75 points or a yield of 6.25 percent. The trader takes a long position and buys one contract. This example is summarized in Table 9.5.

Suppose that on December 15, the December T-bill contract is trading at 92.75 points and the yield on T-bills is 7.25 percent. The yield implied by the futures price is also 7.25 percent. The trader offsets the long position and sells the futures contract. The net on the futures trade is –100 basis points or –\$2,500. The manager pays \$981,875 for a 90-day T-bill with a par value of \$1 million, delivers the T-bill for \$984,375, and uses the gains in the cash market to offset losses in the futures. The net cost of the delivered T-bill is \$984,375. The annualized yield on the T-bill is therefore 6.25 percent. The trader makes the anticipated small profit.

In both of the examples, the traders took long positions in the futures market to protect their cash market positions. The traders will lose money, or will make less than expected, if the interest rates decrease. Thus, both traders are short the cash market—the cash market here being the market for the actual debt instruments—and therefore need to take long positions in the futures to protect that cash position.

Note that basis error and basis risk are not present in either example. We are not referring to the basis points relevant to interest rate changes. Rather, we are referring to the difference between the price of the debt instrument and the price of the futures contract for that instrument. Basis error and basis risk are generally not present when you are hedging the exact instrument that underlies the futures contract. Here we are hedging T-bill purchases and sales with T-bill futures. As with currencies and indexes, basis error is essentially zero in the futures contract expiration month. These markets for debt instruments are the most liquid of any market, and it is very inexpensive to deliver the debt instruments called for in the futures contract. These instruments are delivered through electronic communication, and the transactions costs are small. *This is quite different from delivery of physical commodities.* Two things, however, are more important than basis error and basis risk for interest rate contracts. These will be discussed following the next example.

The following is an example hedge in which the trader sells futures contracts. An agribusiness subsidiary of a large corporation has experienced cost overruns on a construction project. The firm will need to borrow \$10 million in six months to complete the project. It is December 15, and the firm anticipates borrowing on June 15. The

Date	Cash Market	Futures Market
September	Contract to deliver T-bill with \$1 million par value in December for \$984,375. Expected yield of 6.25%.	Buy one December T-bill contract at 93.75 points. Implied yield is 6.25%.
December	Purchase T-bill with \$1 million par value for \$981,875. Deliver T-bill. Receive \$984,375. Gain of \$2,500.	Sell one December T-bill contract at 92.75 points. Loss of \$2,500.
	Net change of \$0. Actual profit equals expected profit.	

TABLE 9.5

Hedging the Future Delivery of Treasury Bills by a Securities Trader

negotiated loan rate is the LIBOR plus 2 percent. However, the interest rate on the loan is not fixed until the funds are borrowed. It is anticipated that the construction will take place, and that this loan will be needed for three months after June 15. After construction is complete, this loan will be paid off and rolled into a larger loan package that is the responsibility of corporate headquarters. The financial officer for the agribusiness firm in charge of the project is concerned about interest rates increasing between December and June, increasing the cost of his part of the project. The June Eurodollar contract is trading at 94.50 points, or a yield of 5.5 percent. The financial officer should be able to lock in a loan rate of 7.5 percent and sells 10 June Eurodollar contracts. Table 9.6 provides the actions taken and the results in terms of the loan rate.

Suppose on June 15 the June contract is trading for 92.50 points. The loan is secured at 9.5 percent. The hedge is lifted, the net change in the futures price is +200 basis points, and the futures position returns \$50,000. The present value of the \$10 million loan is \$9.775669 million.

$$\$10 \text{ million} / (1.095)^{0.25} = \$9.775669 \text{ million.}$$

The present value of the loan would be \$9.820823

$$\$10 \text{ million} / (1.075)^{0.25} = \$9.820823 \text{ million}$$

if it could have been secured at 7.5 percent the previous December. The loss to the cash position is \$45,154. Gains in the futures market are used to offset the increased costs of the loan.

This hedge presented three concepts related to interest rate hedging. The first is basis. Again, it is not the basis points that are relevant to interest rate changes. Rather, the loan rate is 2 percent over the Eurodollar rate. This is very familiar to traders of physical commodities and should pose no problem. In the example, the trader recognized that the expected interest rate for the loan is the rate implied by the futures contract price plus the basis. Further, the actual basis equaled the expected basis. This is because the 2 percent basis was contracted with the loaning bank. Suppose the financial officer calculated the 2 percent premium from historical data. If the actual basis in June is 2.5 percent over the LIBOR, then the loan will be secured at 10 percent. In this case, the present value of the loaned amount is \$9.764541 million

$$\$10 \text{ million} / (1.10)^{0.25} = \$9.764541 \text{ million}$$

TABLE 9.6

Hedging a Future 90-Day Loan Based on the LIBOR Plus 2%

Date	Cash Market	Futures Market
December	Anticipate borrowing funds with \$10 million par value in June for the LIBOR plus 2% or 7.5%. Market value of loan is \$9,820,823.	Sell 10 June Eurodollar contracts at 94.50 points. Implied yield is 5.5%.
June	Borrow \$10 million for LIBOR plus 2% or 9.5%. Market value of loan is \$9,775,669. Loss of \$45,154.	Buy 10 June Eurodollar contracts at 92.50 points. Implied yield is 7.5%. Gain of \$50,000.
	Net change of +\$4,846. Actual loan rate of 7.29%.	

Date	Cash Market	Futures Market
December	Anticipate borrowing funds with \$10 million par value in June for the LIBOR plus expected basis. Expected basis is 2% so expected loan rate is 7.5%. Market value of loan is \$9,820,823.	Sell 10 June Eurodollar contracts at 94.50 points. Implied yield is 5.5%.
June	Borrow \$10 million for LIBOR plus actual basis of 2.5% or 10%. Market value of loan is \$9,764,541. Loss of \$56,282.	Buy 10 June Eurodollar contracts at 92.50 points. Implied yield is 7.5%. Gain of \$50,000.
	Net change of -\$6,282. Actual loan rate of 7.78%.	

TABLE 9.7

Hedging a Future 90-Day Loan Based on the LIBOR

and the loss in the cash market is -\$56,282. This variation on the example is shown in Table 9.7.

Basis error and basis risk is present if you are not hedging the exact instrument underlying the futures contract or if the basis level is not contracted and it is calculated from historical data. Basis risk should be relatively small because the markets for debt instruments are well linked. But the premiums and discounts associated with debt instruments do change relative to the debt instrument interest rates traded on the futures market. In our earlier example, the premium over the LIBOR could have increased from 2 percent to 2.5 percent because business conditions changed during the six months over the hedge. In June it is perceived that short-term construction loans are more risky, and there is an increase in the basis premium.

As in markets for physical commodities, estimates of basis are essential for operating hedging program for debt instruments. Two things are also important for hedging with interest rate contracts. The first is the lumpiness of the contracts. Like exchange rate futures and stock index futures, these futures contracts involve large amounts of financial instruments, as shown in Table 9.1. However, the larger the debt instrument that needs to be hedged, the less this is an issue. But it can be important and almost always keeps gains or losses in the futures market from exactly offsetting losses or gains in the market for the debt instrument.

The second concern that arises through these examples is in matching the cash position with an appropriate position in the futures market. There are two pieces to the puzzle of defining an appropriate futures market position: (1) what particular contract and (2) how many contracts are necessary for an effective hedge? This concept will be covered extensively in Chapter 10, Index Futures Contracts. The same issue arises for hedging interest rate risk. The trader used 10 Eurodollar contracts in the preceding example. The loan was based off the LIBOR and the maturity is 90 days so the Eurodollar is the right contract. But is 10 the right number of contracts? The returns in the futures market offset losses in the cash when there was no basis risk, but not exactly. Did the financial officer hedge too much, or was the difference because the size of the cash position does not match up perfectly with a number of futures contracts?

The hedger can go about matching the cash position with the correct futures position four ways. First, at a minimum, the hedger should attempt to match the par value of the two positions. This is the *par value approach* and is what the financial

officer did in the earlier examples. The number of contracts is the ratio of the par value of the debt instrument in the cash market to the par value of the futures contract.

$$\text{Hedge Amount} = \frac{\text{Par Value Cash Instrument}}{\text{Par Value 1 Futures Contract}}$$

Further, the hedger should choose a futures contract in which the maturity of the underlying debt instrument is similar to the maturity of the debt instrument being hedged. But if there is no exact match, the approach does not provide a correction for this. The approach does not account for compounding differences or maturity differences between the instruments.

The second approach addresses another major weakness of the first approach. The par values of the debt instruments are not traded. It is the market values of the debt instruments that change with changing interest rates. The *market value approach* uses the ratio of the market value, or the price, of the cash debt instrument relative to the market value of the debt instrument underlying the futures contract.

$$\text{Hedge Amount} = \frac{\text{Market Value Cash Instrument}}{\text{Market Value 1 Futures Contract}}$$

Returning to our example, the market value of the \$10 million loan for 90 days at 7.5 percent is \$9.820823 million.

$$\text{\$10 million} / (1.075)^{0.25} = \text{\$9.820823 million}$$

The market value of a Eurodollar contract trading at a 5.5 percent yield is

$$\text{\$1 million} (1 - 0.055(90/360)) = \text{\$0.98625 million}.$$

The ratio, which gives us the number of contracts, is

$$\text{\$9,820,823} / \text{\$986,250} = 9.96 \text{ contracts}.$$

Thus, we see that the financial officer was slightly overhedged in trading 10 contracts. This is one cause of the imbalance between gains and losses in the cash and futures market. But, because of the lumpiness of the contracts, the officer can do nothing about this.

The third possibility is the *basis point approach*. The hedger's real objective is likely to match gains and losses in the cash and futures markets, and not to match the par value or market value of the two positions. Matching the market values will come closer to matching gains and losses than matching the par values, but the hedge still may not be totally effective. The hedger can do a better job of matching gains and losses in the cash and futures markets by comparing the results of a one-basis-point change in the cash debt instrument to the futures contract. The hedge amount is

$$\text{Hedge Amount} = \frac{\text{Value 1 Basis Point Change Cash Instrument}}{\text{Value 1 Basis Point Change Futures Contract}}$$

Again, using our example, a one-basis-point increase in the Eurodollar contract with a \$1 million par value is worth \$25. A one-basis-point increase in a \$10 million loan for 90 days at 7.5 percent is worth \$228. The ratio of the change in the value of the cash instrument to the change in the value of one futures contract is

$$\$228 / \$25 = 9.12.$$

This suggests that the officer needs to trade nine Eurodollar contracts. The officer is slightly overhedged using the market value approach because the loan rate is at a premium to the LIBOR. A one-basis-point change in the LIBOR has a slightly smaller impact on the loan market value than the Eurodollar market value.

While the financial officer in our example is overhedged according to the basis point approach, the magnitude is small and we could ignore it. However, if we modify the example slightly, the basis point approach shows its strength. Suppose the firm requires the loan for one year instead of 90 days, all of the other terms of the loan being identical. Now, a one-basis-point change in the interest rate costs the firm \$866. This is calculated from

$$\$10 \text{ million} / (1.0750)^1 = \$9,302,326 \text{ million}$$

and

$$\$10 \text{ million} / (1.0751)^1 = \$9,301,460 \text{ million}.$$

The ratio of the cash amount to the futures is 34.4 (\$866/\$25). The method suggests the officer needs to trade 34 Eurodollar contracts. You see that the officer needs to trade more contracts of a debt instrument with a 90-day maturity to hedge a cash debt instrument (i.e., the loan) with a one-year maturity. Table 9.8 presents an example outcome of a hedge following this recommendation.

Now, suppose the loan is not priced off the LIBOR but is priced off the prime rate. We can calculate the change in the market value of the loan given a one-basis-point change in the prime rate. It will be the same as a one-basis-point change in the LIBOR if the prime rate is equal to the LIBOR plus 2 percent. What we really need to know is how the prime rate changes relative to the LIBOR. The prime is more variable, and we will assume the prime rate is 25 percent more variable for the example. Thus, a 100-basis-point change in the LIBOR will be accompanied by a 125-basis-point change in the prime. The basis approach ratio can be augmented by a factor that captures the relative variability of the two interest rates.

Date	Cash Market	Futures Market
December	Anticipate borrowing funds with \$10 million par value in June for the LIBOR plus 2% or 7.5%. Market value of loan is \$9,302,326.	Sell 34 June Eurodollar contracts at 94.50 points. Implied yield is 5.5%.
June	Borrow \$10 million for LIBOR plus 2% or 9.5%. Market value of loan is \$9,132,420. Loss of \$169,906.	Buy 34 June Eurodollar contracts at 92.50 points. Implied yield is 7.5%. Gain of \$170,000.
	Net change of +\$94. Actual loan rate of 7.50%.	

TABLE 9.8

Hedging a Future One-Year Loan Based on the LIBOR Plus 2%

$$\text{Hedge Amount} = \frac{\text{Value 1 Basis Point Change Cash Instrument}}{\text{Value 1 Basis Point Change Futures Contract}} \times \text{Relative Variability.}$$

Returning to our example, suppose the prime rate is about equal to the LIBOR plus 2 percent, so a one-basis-point change in the loan rate still costs the firm \$866. Next, suppose the prime is 25 percent more variable than the LIBOR. The financial officer should trade

$$(\$866 / \$25) \times 1.25 = 43.3$$

or 43 Eurodollar contracts. You also see that the financial officer needs to trade more contracts of a debt instrument that is less variable than the cash debt instrument (i.e., the loan) that you are trying to hedge. Table 9.9 presents an example outcome of a hedge following this recommendation.

How do you calculate the relative variability? The tool used is the same as that used to measure the relative variability in returns of different stock portfolios and is discussed in many finance texts. This tool will be discussed in more detail in Chapter 10. The relative variability between two interest rates is measured with the following linear model:

$$CMR_t = \alpha + \beta FMR_t + e_t$$

where CMR_t denotes the interest rate for the cash market instrument time t , FMR_t denotes the interest rate for which we have a futures contract during the same time period t , α and β are parameters to be estimated with the data, and e_t is the random unexplainable error. This model can be estimated with linear regression. All commercial spreadsheet software will perform this calculation. The parameter β measures the relative variability. If β equals 1, then the interest rate for the cash instrument moves one-for-one with the interest rate associated with the futures contract. A 1 percent increase or decrease in the futures market rate will, on average, be matched by a 1 percent increase or decrease in the cash instrument. If β is less than 1, the interest rate of the cash instrument is less variable than the futures market. If β is more than 1, the interest rate of the cash instrument is less variable than the futures market. In addition to the measure of relative variability, the model also provides an estimate of the average basis premium or discount through the estimate α .

TABLE 9.9

Hedging a Future One-Year Loan Based on the Prime Rate

Date	Cash Market	Futures Market
December	Anticipate borrowing funds with \$10 million par value in June at prime. Expect prime to be LIBOR plus 2% or 7.5%. Market value of loan is \$9,302,326.	Sell 43 June Eurodollar contracts at 94.50 points. Implied yield is 5.5%.
June	Borrow \$10 million at prime. Prime is LIBOR plus 2.5% or 10%. Market value of loan is \$9,090,909. Loss of \$211,417.	Buy 43 June Eurodollar contracts at 92.50 points. Implied yield is 7.5%. Gain of \$215,000.
	Net change of +\$3,583. Actual loan rate of 7.46%.	

The last procedure is the *duration approach*. This approach more completely accounts for differences in the maturity of the cash debt instrument and the futures contract used to hedge it. The hedge amount is calculated as the following

$$\text{Hedge Amount} = \frac{(1 + r_f) \times P_c \times T_c}{(1 + r_c) \times P_f \times T_f} \times \text{Relative Variability}$$

where P_c is the market value of the cash instrument, P_f is the market value of the underlying futures instrument, T_c is the maturity of the cash instrument, T_f is the maturity of the underlying futures instrument, r_c is the interest rate of the cash instrument, and r_f is the interest rate of the futures instrument.

Let's calculate the hedge amount for the 90-day \$10 million loan.

$$\text{Hedge Amount} = \frac{(1 + 0.055) \times \$9,820,823 \times 0.25}{(1 + 0.075) \times \$986,250 \times 0.25} = 9.77$$

or the method suggests ten contracts. The hedge amount for the one-year \$10 million loan is

$$\text{Hedge Amount} = \frac{(1 + 0.055) \times \$9,302,326 \times 1}{(1 + 0.075) \times \$986,250 \times 0.25} = 37.03$$

or the method suggests 37 contracts. And if the loan is priced off the prime rate, which is 25 percent more variable than the LIBOR,

$$\text{Hedge Amount} = \frac{(1 + 0.055) \times \$9,302,326 \times 1}{(1 + 0.075) \times \$986,250 \times 0.25} \times 1.25 = 46.28$$

or 46 contracts.

If you look closely at the formula you see the following: The greater the cash interest rate is relative to the futures interest rate, the fewer contracts are traded. The greater the cash instrument market value relative to the futures instrument market value, the more contracts are traded. And the greater the cash instrument maturity relative to the futures instrument maturity, the more contracts are traded. Thus, this approach corrects for differences in the market value between the cash and futures instruments, differences in maturity of the two instruments, and differences in the level of the underlying interest rates.

We will finish this section with two more example hedges. The first example is known as a strip hedge. The structure of the loan appears strange but the procedures of this hedge are useful in understanding the second example, converting a floating rate loan to a fixed rate loan. In the example strip hedge, a firm borrows \$1 million at 3 percent over the LIBOR for three months, pays the interest after three months, rolls the loan over for a second three-month period, and then follows the same procedure for a third and fourth three-month period. Basically, the firm is borrowing the money for one year and making quarterly interest payments. The initial loan is taken out in December. The firm hedges the loan by selling one Eurodollar future for each one on the March, June, and September contracts. When the loan is refinanced in March, the firm buys back the March contract. When the loan is refinanced in June, the June con-

TABLE 9.10

Strip Hedge of a One-Year Balloon-Payment Loan Based on the LIBOR Plus 3%

Date	Cash Market	Futures Market
December	Borrow \$10 million at 8.75%. Commit to roll over loan for three quarters at prevailing LIBOR plus 3%. Expected interest cost is \$888,750 based on expected yield of 8.8875% which is average of current rate and implied rates from futures.	Sell 10 March Eurodollar contracts at 94.15 points. Sell 10 June Eurodollar contracts at 94.05 points. Sell 10 September Eurodollar contracts at 94.00 points.
March	Pay interest of \$218,750. Roll over \$10 million loan for three months at 7.85%.	Buy 10 March Eurodollar contracts at 95.15 points. Loss of \$25,000.
June	Pay interest of \$196,250. Roll over \$10 million loan for three months at 9.95%.	Buy 10 June Eurodollar contracts at 93.05 points. Gain of \$25,000.
September	Pay interest of \$248,750. Roll over \$10 million loan for three months at 10.5%.	Buy 10 September Eurodollar contracts at 92.50 points. Gain of \$37,500.
December	Pay interest of \$262,500 and repay principal. Total interest cost is \$926,250.	Total gain of \$37,500.
Net interest cost is \$888,750.		

tract is bought back, and likewise for the September contract. Currently, the March Eurodollar contract is trading at 94.15, June is trading at 94.05, and September is at 94.00. The firm should be able to lock in interest payments of 8.85 percent, 8.95 percent, and 8 percent. The interest payment for the first quarter is set when the loan was initiated. This example is summarized in Table 9.10.

The interest payment for the first quarter is set when the loan is initiated. However, the interest payment falls for the second quarter, and then rises for the next two. Through the hedge, the gains and losses in expected interest payments are offset by gains and losses in the futures market.

In the second example, a hedger converts a floating rate loan to a fixed rate loan. It is currently December, and an agribusiness exporter needs to borrow \$100 million for one year. The firm will make quarterly payments to interest and principal, and a new interest rate is determined each quarter by LIBOR plus 3 percent. This hedge is different from the previous example, the strip hedge, in that the firm is less exposed to interest rate risk towards the end of the loan since principal is being repaid. The firm sells 74 Eurodollar futures in the March contract, 50 June contracts, and 25 September contracts. The firm lifts portions of the hedge components as the loan is repaid. The prices are 94.15, 94.05, and 94.00. The firm should be able to lock in interest payments of 8.85 percent, 8.95 percent, and 9 percent. Payments for the first quarter are based on 8.75 percent. This example is summarized in Table 9.11.

Date	Cash Market	Futures Market
December	Borrow \$100 million for one year at LIBOR plus 3%. Current rate is 8.75%. Expected interest cost is \$4.8522 million based on expected yield of 8.8875% which is average of current rate and implied rates from futures.	Sell 74 March Eurodollar contracts at 94.15 points. Sell 50 June Eurodollar contracts at 94.05 points. Sell 25 September Eurodollar contracts at 94.00 points.
March	Interest paid is \$2.012 million. Current rate is 9.85%.	Buy 74 March Eurodollar contracts at 93.15 points. (+100 basis points) Gain of \$185,000.
June	Interest paid is \$1.6656 million. Current rate is 10.45%.	Buy 50 June Eurodollar contracts at 92.55 points. (+150 basis points) Gain of \$187,500.
September	Interest paid is \$1.1165 million. Current rate is 7.5%.	Buy 25 September Eurodollar contracts at 95.50 points. (–150 basis points) Loss of \$93,750.
December	Interest paid is \$0.3236 million. Total interest cost is \$5.1177 million.	Total gain of \$278,750.
Net interest cost is \$4.8390 million.		

TABLE 9.11.
Converting a Floating
Rate Loan to a Fixed
Rate Loan

The interest rate increases, effecting payments in the second and third quarter. The interest rate then declines in the fourth quarter. Increases in interest rates early in the repayment of the loan will create a problem for the firm. The amount of principal to be repaid on the loan is high at this point. Likewise, the decline in the interest rate later in the loan is comparatively not much of a benefit because most of the principal has been repaid. Thus, to hedge the loan, more nearby contracts are sold relative to deferred contracts. And, as the example shows, the gains and losses in the cash market are offset by gains and losses in the futures market.

The interest rate futures have a “matching” issue that is not always present in physical commodities. But the techniques are not difficult, and the basic idea behind the hedge is still the same: protect against risk.

INTEREST RATE FUNDAMENTALS

Interest rate fundamentals are probably the simplest of all those financial instruments for which futures contracts are traded. This is not to say that interest rates are easy to predict. Rather, the factors that influence the level and changes in interest rates are reasonably well known. While these factors remain difficult to predict, understanding

what factors influence interest rates and where this information may be gathered will allow you to understand interest rate behavior.

Interest is the price paid by borrowers to lenders of money and capital. And like any price, the balance between supply and demand determines the equilibrium level, and changes in supply and demand factors will result in changes in the equilibrium price. If the supply of money and capital increases, the price will fall. If the demand for money and capital increases, the price will rise.

There is an anticipated long-run rate of return in the capital market. Borrowers of capital anticipate putting that capital to productive use and paying the lender principal plus interest. Lenders of capital are willing to delay current consumption for higher rates in the future, or are interested in spreading consumption of current wealth out over many periods. Money and capital markets allow both borrowers and lenders to pursue these goals. The long-run rate of return to money and capital depends on the productivity of borrowers and the preferences of lenders between consuming wealth now or some time in the future. This long-run rate of return is present in debt instruments with different maturities.

On a much less abstract level, the factors that determine the level and changes in interest rates come from four broad areas: business conditions, household conditions, fiscal policy, and monetary policy. The institutions in these areas make use of capital, have capital to loan, or influence the intermediaries in capital markets. We will address each of these areas, briefly, in turn. Within this framework, interest rates generally follow the broader business and economic cycle. There is a long-run rate of return paid and received on borrowed and loaned capital. Market-determined interest rates cycle around this rate of return as the economy moves from periods of excess demand to periods of excess supply. A strong and growing economy is usually constrained by production capacity and timing limitations. Expanding production requires money and capital and generally leads to increased interest rates. The central bank may also increase base interest rates to slow the economy and prevent inflation. Economic downturns always follow expansions. At some point, the economy turns from excess demand to excess supply. A weak economy has excess capacity and inventories. These conditions result in decreased demand for money and capital and lead to decreased interest rates. Likewise, the central bank will decrease base interest rates after inflation has been reduced to economic growth. Then the cycle repeats.

Businesses are the largest users of long-term capital, and are the largest providers and users of short-term capital. A strong and growing economy will lead to the increased need for business to borrow money and capital. If all else is constant in this market, then this demand will put upward pressure on interest rates. Specific components to watch within the area of business conditions include construction activity and inventories. Businesses will need to finance construction of plants and other facilities, and will need to finance the operating cost necessary to replenish shrinking inventories. Production, employment, overall price levels, and business profits are good indicators of business conditions. Increasing production, represented in Gross Domestic Product; increasing employment and decreasing unemployment; increasing retail, wholesale, and raw material prices; and increasing business profitability are all signals of a strong economy that must be matched with increasing demand for capital and the possibility of increasing interest rates. Statistics on these various factors are reported in publications released from the U.S. Department of Commerce and the Federal Reserve System.

Consumers are the other side of the economic coin. *Demand by consumers for goods and services is the primary factor leading to a strong economy.* The con-

sumer spends the first dollar and then all businesses compete for a portion of that dollar. Initial spending by consumers leads to spending by retail firms, marketing firms, and initial producers. Strong consumer demand is synonymous with a strong economy. Likewise, weak economies are accompanied by weak demand.

Employment, personal income, and consumer credit conditions are indicators of consumer demand. Increasing employment, increasing personal income, and low consumer debt are precursors to expanded consumer spending. Likewise, increasing unemployment, decreasing real personal income, and high consumer debt will lead to decreases in consumer spending. Again, the U.S. Department of Commerce and Federal Reserve System compile these data and release publications with this information.

Consumers demand a significant portion of short-term credit, but they are the largest providers of long-term capital. This long-term capital is available through retirement savings. Short-term borrowing is usually largest during economic downturns, lessens at the bottom of the business cycle, and increases at the beginning of an economic upturn. Long-term capital is most available after sustained economic growth. The need for borrowing or provision of capital to loan by consumers generally accentuate conditions in the money and capital markets.

Government is one of the largest users of intermediate-term capital and is a significant borrower of short-term capital. Government capital needs are driven by fiscal policy. Fiscal policy is the combination of all spending on military, social programs and public works, net of revenue from tax collections. All else being constant, increases in government spending relative to tax receipts requires the financing of a deficit and will increase the interest rate, and decreases in spending relative to tax receipts will allow deficit reduction and will decrease the interest rate.

During the 1980s, the U.S. federal government borrowed heavily. The impact on interest rates was apparently rather limited because of the strong demand for U.S. government securities overseas. While current deficit and debt conditions appear to be improving, more deficit and debt financing appear to be needed in the future. Currently, the U.S. federal government has significant obligations in terms of future payments to the Social Security and Medicare programs.

We've saved the biggest and best for last. Monetary policy, as implemented by the U.S. Federal Reserve Bank, is one of the most important if not the most important factor influencing interest rates. The main objective of the Federal Reserve Bank is to promote stable economic growth. Economic growth can be increased or decreased by changing the money supply and by controlling the availability of credit. The Federal Reserve Bank (the Fed) carries out monetary policy through three main venues.

The Fed buys and sells U.S. government securities, treasury bonds, notes, and bills, through *open market operations*. When the Fed buys government securities, this increases the supply of money in bank reserves. Banks are able to loan this money. When the Fed sells government securities, this decreases the supply of money in bank reserves. Increases and decreases in the money supply through open market operations increase and decrease loanable funds in the banking system, and should result in decreases and increases in interest rates. Changes in the money supply in this fashion rapidly affect many financial institutions. The Fed used open market operations to increase the money supply following the stock market "corrections" of 1983, 1987, and 1997. The availability of these funds enabled many different financial institutions to meet their short-term obligations without excessive long-term problems associated with their limited cash reserves.

The Fed determines *reserves requirements* that must be kept by banks. Reserve requirements are the percent of deposits that every bank must keep to satisfy regula-

tions and be insured through government sources. Reserve requirements can be changed to induce a change in interest rates. Increasing reserve requirements increases the reserve funds that each bank must hold, which decreases the money supply in capital and money markets and, in turn, will lead to an increase in the interest rate. This monetary tool is used infrequently when compared to open market operations.

There is also a market for these reserve funds or *federal funds*. Banks with reserves in excess of requirements loan these funds to banks with insufficient reserves. The federal funds rate is an important indicator of short-term interest rates and sustained changes in this rate often lead to changes in interest rates for longer-term debt instruments. The CBOT also trades a futures contract for 30-Day Federal Funds.

One method that banks use to increase their cash reserves is to borrow money from the Fed. Banks will borrow money from the Fed to loan in other markets if they have insufficient deposits to meet demands for funds. The interest rate on loans from the Fed is the *discount rate*. The discount rate is very much the interest rate floor in the market for short-term debt. Banks borrow money from the Fed and then mark up this rate to reflect the bank costs before loaning it again. Bank costs include the risk of default by borrowers. The discount rate is set by the Board of Governors of the Federal Reserve System. Lowering the discount rate will lower interest rates, particularly for instruments with short maturities, through the entire banking system. Increasing the discount rate will increase interest rates.

You will be helped if you remember that the interest rate is the price of money. Supply and demand interact to discover price, the same process you went through in earlier chapters dealing with agricultural commodities. Since the interest rate is a price, this helps you see why the interest rate changes when federal agencies release information showing strong demand for money (housing starts are up, for example) or when the Fed changes the supply of money available to the business world.

SUMMARY

Interest rate futures contracts are the most successful of all futures contracts. The volume of trade in futures contracts related to interest rates is the largest of all the categories of futures.

Interest rate futures contracts involve trading the market value of a debt instrument. Debt instruments are financial products that yield the holder either a one-time payment or a stream of interest payments. With short-term debt instruments, the investor buys the instrument at a market value which is less than the par value. At maturity, the investor redeems the instrument for the par value. The difference is the interest payment. With long-term debt instruments, the investor purchases the instrument at a market value. The market value may or may not be less than the par value. This is because the investor also receives a stream of interest payments between purchase and maturity. While the business or government that issues the instrument is responsible for paying the holder the par value at maturity, almost all debt instruments are traded more than once. As the market value, or price, of debt instruments with fixed par values (and possibly streams of interest payments) changes in this secondary market, the implied interest rate fluctuates. Futures exchanges have developed contracts that allow traders in this market to manage their risk exposure.

The different exchanges have developed futures contracts for different products. The CME focuses on short-term debt instruments; these include treasury bills and Eurodollar deposits. The interest rate underlying Eurodollar contracts is the London Interbank Offered Rate (LIBOR). The CBOT trades futures on long-term debt instruments—treasury bonds with 15 years to maturity and treasury notes with 10, 5, and 2 years to maturity.

Trading in these contracts is not a trivial matter. It requires some familiarity with bond pricing. Short-term instruments are the easiest to understand. The instrument is traded at some discount to the par value. The price of the futures contract is this discount. Thus, if the discount increases, the implied interest rate decreases. A trader that wants to sell the interest rate because it is expected to decline needs to buy the futures contract. Long-term instruments are more difficult. The instrument is also traded at a discount or premium to the par value. Thus, as with short-term instruments, the trader that wants to sell the interest needs to buy the futures contract. The difficulty is with the units. Long-term instruments are quoted in points and 32nds, and it is difficult to quickly convert this into an annual percent interest rate.

Hedging debt instruments are similar but also more difficult than other financial instruments. As with hedging stock portfolios, the trader needs to be careful to match the cash position with the proper futures position. There are a number of ways to do this. The more accurate methods entail increasing complexity. The procedures discussed in this chapter include the par value, market value, basis point, and duration approaches. The first two are the simplest and the least accurate. The third approach attempts to match changes in the value of the cash position with a futures position that offsets those changes. The fourth approach is similar to the third, but the approximation is better because the method accounts for the different maturities between the cash instrument and the instrument underlying the futures contract.

We reiterate, a crucial part of hedging interest rates involves matching the cash and futures positions so that for changes in market interest rates, changes in the value of the cash position are offset by changes in the value of the futures position. The hedger must account for differences in market value, changes in the market value with changes in different interest rates, and differences in the maturity of the cash and futures instruments.

Basis error is also present when hedging interest rates. Money and capital markets are as well integrated as exchange rate and stock index cash and futures markets, but interest rate markets must price risks not present in these other markets. Because of this, the premiums and discounts across different interest rates can change in unpredictable ways.

As with prices for other commodities, the price of money and capital is determined by the supply of and demand for money and capital. Interest rates, the price of capital, will increase if the demand for capital is high relative to its availability. Conversely, interest rates will fall if capital is readily available given the level of demand. In addition to market forces, the Federal Reserve Bank is an institution with instruments at its disposal that can influence the supply and demand for capital. The overall goal of the Federal Reserve Bank is stable economic growth, and to achieve this the bank will change the money supply and reserve requirements. Capital markets react accordingly.

With the continued industrialization, integration, and progress in global capital markets, the volume of trade in money and capital markets will continue to grow. *The need for futures markets with which to manage risk will likely grow as well.*

KEY POINTS

- *Interest rate futures contracts* are the most successful futures contract. Not all of the interest rate futures contracts that have been introduced have been successful, but there have been *tremendous successes*.
- Interest rate futures contracts involve *trading the market value of a debt instrument*.
- *Debt instruments are financial products* that yield the investor interest payments. *Short-term debt instruments* are purchased at market value, redeemed at par value, and the difference is the interest payment. *Long-term debt instruments* are purchased at a market value, redeemed for par, and receive coupon payments prior to maturity.
- Debt instruments are *traded in secondary markets*—not just between the issuer and first buyer. As the market value of debt instruments with fixed par value and interest returns changes, *the implied interest rate fluctuates*. Futures contracts allow traders to *manage this risk*.
- The CME trades futures on *short-term debt instruments*: treasury bills and Eurodollars. The CBOT trades futures on treasury bonds with *15 years to maturity*, and treasury notes with *10, 5, and 2 years to maturity*.
- Short-term instruments are traded at *discount to par value*. This is the *futures price*. If the discount increases, the *interest rate decreases*. Long-term instruments are also traded at a discount or premium to the par value. However, the units are points and 32nds of par. If the discount increases, the *interest rate decreases*.
- Hedging debt instruments is similar *but more difficult* than other financial instruments. The cash position needs to be *correctly matched with the proper futures position*. Par value, market value, basis point, and duration approaches were discussed. The hedger needs to account for *differences in market value*, changes in that value with *changes in different interest rates*, and *differences in the maturity of the cash and futures instruments*.
- *Basis error* is present when hedging interest rates. The premiums and discounts across different lending and borrowing rates *can change in unpredictable ways*. These changes reflect risks *not necessarily captured* in the price of the debt instrument traded in the futures contract.
- Interest rates are determined like any other *price*. Interest rates, *the price of capital*, will increase if the demand for capital is high relative to supply. And *interest rates will fall* if the demand for capital is low relative to supply. However, the Federal Reserve Bank plays a large role in *determining the interest rate*. The Federal Reserve Bank *controls money supply*, and capital markets react quickly to changes in money supply.

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