COLORADO STATE UNIVERSITY

Assignment 1 **Agricultural & Resource Economics / Economics 535** *S.R. Koontz*
Fall 2023 **Applied Econometrics**

This assignment is worth 25 points. This work is to be an independent effort on your part. Please show your work concisely. Printed spreadsheet pages are sufficient for calculations. A handwritten document is sufficient for the other questions. Correct answers that are disorganized and do not have clear supporting discussion are worth little. Communication is important. And so is clarity.

1. Answer exercise 2.9 in Gujarati & Porter.

2. Answer exercise 3.11 in Gujarati & Porter. Do not use sample notation: $\sum_{i=1}^{N} \frac{X_i}{N}$. Use expectation notation: E(X) – also V(X) and Cov(X,Y).

3. Answer exercise 3.17 in Gujarati & Porter.

4. Refer to exercise 3.23. Using ln(RGDP) as Y and X as defined in the exercise, construct a table that is similar to Table 3.2 and that includes the notes at the bottom. (One printed page for the spreadsheet and results in a clearly labeled table.) Use the data provided and not the data in the textbook. Be careful to define Y = ln(RGDP). Also, there are some typos in the table notes that are correct in the text. Refer to the text.

5. Using the data from question 4, also construct: the correlation of Y and X, the correlation of y and x, the mean of Y and X, the variance and standard deviation of Y and X, and the predicted value of Y for the year 2023. (One year out of sample.)

6. Conduct a two-tailed test of intercept and slope coefficient estimates. Use an alpha level of 5%. Conduct a one-tailed test of the slope coefficient as in the notes. Calculate the p-value for the intercept and slope coefficients. Calculate a 90% confidence interval for the error variance.

7. What are the variance, and square root of, for the predicted value of Y? (This is needed for a confidence interval.)

8. What is the estimate of the error variance, and square root of, using the maximum likelihood estimator? Likewise, what are the variances, and standard errors, of the intercept and slope coefficients?

9. Answer exercise 5.5 in Gujarati & Porter.

10. Optional: Answer exercise 4.3 in Gujarati & Porter.

1. (2.9)

$Y_i = \frac{1}{\beta_0 + \beta_1 X_i}$ can be estimated by the following linear-in-parameters model $\frac{1}{Y_i} = \beta_0 + \beta_1 X_i$

$Y_i = \frac{X_i}{\beta_0 + \beta_1 X_i}$ can be estimated by $\frac{X_i}{Y_i} = \beta_0 + \beta_1 X_i$

$Y_i = \frac{1}{1 + \exp(-\beta_0 - \beta_1 X_i)}$ can be estimated by $\left(-\ln\left(\frac{1}{Y_i} - 1 \right) \right) = \beta_0 + \beta_1 X_i$

So, the models can be made linear in parameters and estimated, and then the estimates can be used with the nonlinear model…

2. (3.11)

Show r_1 = Corr(Y,X) = Corr(X,Y) = r_2 = Corr(aX + b, cY + d)

$$r_1 = \frac{Cov(X,Y)}{\sqrt{V(X)}\sqrt{V(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

$$r_2 = \frac{Cov(aX+b, cY+d)}{\sqrt{V(aX+b)}\sqrt{V(cY+d)}} = \frac{ac\sigma_{YX}}{ac\sigma_Y \sigma_X} = \frac{\sigma_{YX}}{\sigma_Y \sigma_X}$$

First, Var(aX + b) = a^2Var(X) and Var(cY + d) = c^2Var(Y)

Second, using Cov(X,Y) = E(XY) – E(X)E(Y),

then E(aX + b) = aE(X) + b, E(cY + d) = cE(Y) + d, and

E((aX + b)(cY + d)) = acE(XY) + adE(X) + bcE(Y) + bd,

So Cov(aX + b, cY + d) = ac Cov(X,Y).

Constant shifts do not change measures of variance or covariance. Scaling matters but cancels in the correlation.

3. (3.17)

If $Y_i = \beta_0 + e_i$ then $e_i = Y_i - \beta_0$ and RSS = $\sum_{i=1}^{N}(Y_i - \beta_0)^2$ so minimizing the sum of squared errors yields $\hat{\beta}_0 = \frac{\sum_i y_i}{N} = \overline{y}$ which is the mean.

And $V(\hat{\beta}_0) = \frac{\sigma^2}{N}$ which is the variance of the mean.

RSS is then equal to TSS.

If we add a variable as long as it has some correlation with the dependent variable (Y) then RSS will decrease. If the new variable is not correlated with Y then just report the mean Blue Crab price!

9. (5.5)

Conduct formal t-test on (stock market) Beta and (stock market) Alpha.

Beta is not different from one.

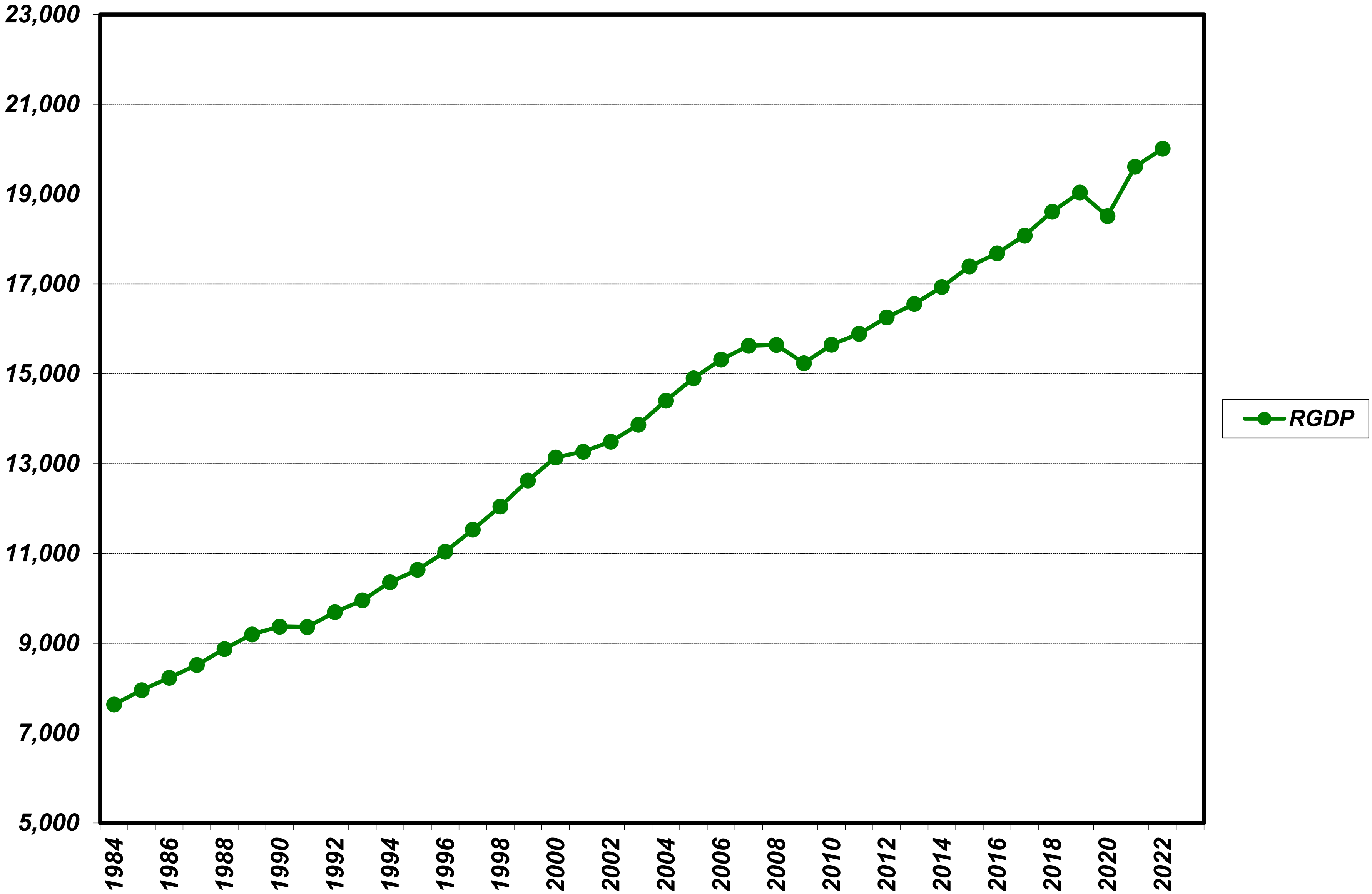
t-stat = (1.0598 – 1) / 0.0728 = 0.8214 and CV 1.9700 or 1.6513 (from MSExcel).
Fail to reject null hypothesis.

Alpha is different from zero.

t-stat = 0.7264 / 0.3001 = 2.4205 with same critical values.
Reject null hypothesis.

Year	RGDP	Y	X	y	x	yixi	x2i	y2i	X2i	Yhati	uhati	uhat2i
1984	7637.7	8.940852	1	-0.541551	-19	10.28947	361	0.293277	1	9.009482	-0.06863	0.00471
1985	7956.2	8.981707	2	-0.500696	-18	9.012527	324	0.250696	4	9.034373	-0.052666	0.002774
1986	8231.7	9.015748	3	-0.466655	-17	7.933133	289	0.217767	9	9.059263	-0.043515	0.001894
1987	8516.4	9.049749	4	-0.432654	-16	6.922459	256	0.187189	16	9.084154	-0.034405	0.001184
1988	8872.2	9.090678	5	-0.391725	-15	5.87587	225	0.153448	25	9.109044	-0.018366	0.000337
1989	9198	9.126741	6	-0.355661	-14	4.979259	196	0.126495	36	9.133935	-0.007193	5.17E-05
1990	9371.4	9.145418	7	-0.336985	-13	4.380804	169	0.113559	49	9.158825	-0.013408	0.00018
1991	9361.3	9.144339	8	-0.338063	-12	4.056759	144	0.114287	64	9.183716	-0.039377	0.001551
1992	9691	9.178953	9	-0.30345	-11	3.337948	121	0.092082	81	9.208607	-0.029654	0.000879
1993	9957.7	9.206101	10	-0.276301	-10	2.763013	100	0.076342	100	9.233497	-0.027396	0.000751
1994	10359	9.245611	11	-0.236792	-9	2.131126	81	0.05607	121	9.258388	-0.012777	0.000163
1995	10636.9	9.272084	12	-0.210318	-8	1.682547	64	0.044234	144	9.283278	-0.011194	0.000125
1996	11038.2	9.309117	13	-0.173285	-7	1.212998	49	0.030028	169	9.308169	0.000948	9E-07
1997	11529.1	9.35263	14	-0.129773	-6	0.778639	36	0.016841	196	9.333059	0.01957	0.000383
1998	12045.9	9.39648	15	-0.085923	-5	0.429615	25	0.007383	225	9.35795	0.03853	0.001485
1999	12623.4	9.443308	16	-0.039095	-4	0.156381	16	0.001528	256	9.38284	0.060467	0.003656
2000	13138	9.483264	17	0.000861	-3	-0.002584	9	7.42E-07	289	9.407731	0.075533	0.005705
2001	13263.4	9.492764	18	0.010361	-2	-0.020722	4	0.000107	324	9.432622	0.060142	0.003617
2002	13488.4	9.509585	19	0.027183	-1	-0.027183	1	0.000739	361	9.457512	0.052073	0.002712
2003	13865.6	9.537166	20	0.054764	0	0	0	0.002999	400	9.482403	0.054764	0.002999
2004	14399.8	9.57497	21	0.092567	1	0.092567	1	0.008569	441	9.507293	0.067676	0.00458
2005	14901.3	9.609204	22	0.126801	2	0.253602	4	0.016078	484	9.532184	0.07702	0.005932
2006	15315.9	9.636647	23	0.154244	3	0.462732	9	0.023791	529	9.557074	0.079572	0.006332
2007	15623.8	9.656551	24	0.174148	4	0.696592	16	0.030328	576	9.581965	0.074586	0.005563
2008	15642.9	9.657772	25	0.17537	5	0.876849	25	0.030755	625	9.606856	0.050917	0.002593
2009	15236.2	9.631429	26	0.149027	6	0.89416	36	0.022209	676	9.631746	-0.000317	1E-07
2010	15649	9.658162	27	0.17576	7	1.230317	49	0.030891	729	9.656637	0.001526	2.33E-06
2011	15891.5	9.67354	28	0.191137	8	1.529096	64	0.036533	784	9.681527	-0.007988	6.38E-05
2012	16254	9.696094	29	0.213692	9	1.923224	81	0.045664	841	9.706418	-0.010323	0.000107
2013	16553.3	9.714341	30	0.231938	10	2.31938	100	0.053795	900	9.731308	-0.016968	0.000288
2014	16932	9.736961	31	0.254558	11	2.800137	121	0.0648	961	9.756199	-0.019238	0.00037
2015	17390.2	9.763662	32	0.281259	12	3.375113	144	0.079107	1024	9.781089	-0.017427	0.000304
2016	17680.3	9.780206	33	0.297804	13	3.871447	169	0.088687	1089	9.80598	-0.025774	0.000664
2017	18076.6	9.802374	34	0.319971	14	4.479592	196	0.102381	1156	9.830871	-0.028497	0.000812
2018	18609.1	9.831406	35	0.349003	15	5.235049	225	0.121803	1225	9.855761	-0.024355	0.000593
2019	19036.1	9.854092	36	0.37169	16	5.947036	256	0.138153	1296	9.880652	-0.026559	0.000705
2020	18509.2	9.826023	37	0.34362	17	5.841548	289	0.118075	1369	9.905542	-0.079519	0.006323
2021	19609.8	9.883785	38	0.401382	18	7.224876	324	0.161108	1444	9.930433	-0.046648	0.002176
2022	20014.1	9.904192	39	0.42179	19	8.014002	361	0.177906	1521	9.955323	-0.051131	0.002614
Sum		369.8137	780	-5.86E-14	0	122.9594	4940	3.135707	20540	369.8137	-7.46E-14	0.075179
Average		9.482403	20	-1.5E-15	0					9.482403		
Var		0.082519	130	0.082519	130					0.08054		
StdDev		0.287261	11.40175	0.287261	11.40175					0.283796		
N or T		39										

4.	B1 =	0.0249	
	B0 =	8.9846	
	S2 =	0.002032	
	v(B1) =	4.11E-07	
	se(B1) =	0.00064	
	v(B0) =	0.000217	
	se(B0) =	0.0147	
	r2 =	0.9760	
	r =	0.9879	
5.	corr(X,Y) =	0.9879	
	corr(x,y) =	0.9879	
	E(Y|X=40) =	9.980214	21595
6.	t(B1) =	38.8106	
	t(B0) =	610.4444	
	df =	37	
	alpha =	0.05	
	t(5%) 2-tail	2.0262	
	t(5%) 1-tail	1.6871	
	p-val(B1) =	1.4E-31	
	p-val(B0) =	1.14E-75	
	Chi(95%)	24.07494	
	Chi(5%)	52.19232	
	S2 Upper	0.003123	
	S2 Lower	0.00144	
7.	v(Yhat) =	0.002248	0.000217
	se(Yhat) =	0.047418	0.004751
	Upper	10.02763	22644
	Lower	9.932796	20595
8.	S2(ML) =	0.001928	0.949
	v(B1) =	3.9E-07	
	se(B1) =	0.000625	0.974
	v(B0) =	0.000206	
	se(B0) =	0.0143	0.974



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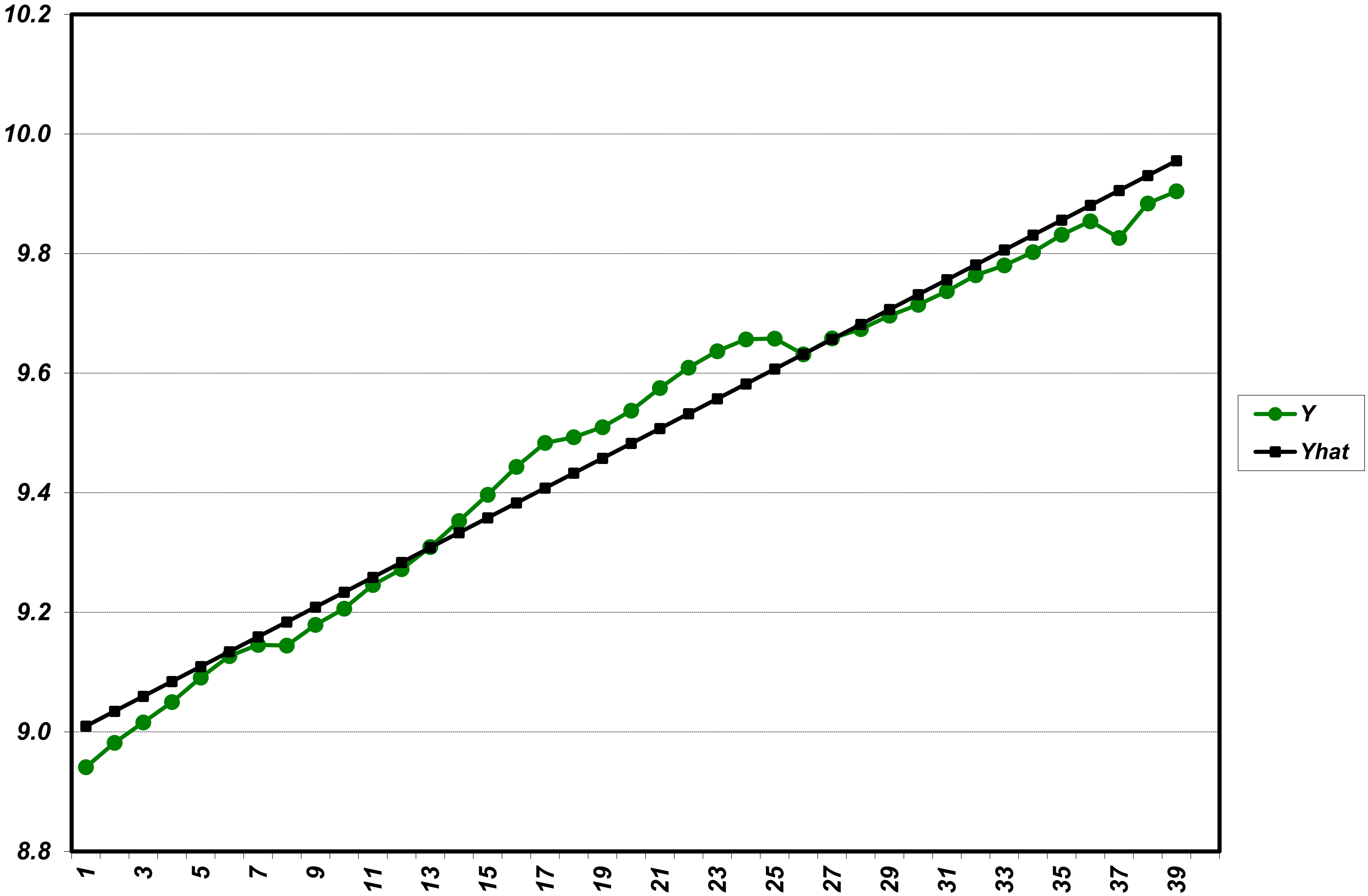
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10. (4.3)

f(X) = $\frac{1}{\theta}e^{-\frac{X}{\theta}}$ and f(X_i) = $\frac{1}{\theta}e^{-\frac{X_i}{\theta}}$

Construct the likelihood function.

L = $\prod_{i=1}^{N}\frac{1}{\theta}e^{-\frac{X_i}{\theta}}$

lnL = $\sum_{i=1}^{N}\left(-\ln\theta-\frac{X_i}{\theta}\right)$

lnL = $-N\ln\theta-\frac{1}{\theta}\sum_{i=1}^{N}X_i$

Find the maximum likelihood estimator for θ.

$$\frac{\partial\ln L}{\partial\theta}=-\frac{N}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^{N}X_i \text{ = set = 0}$$

$$\frac{N}{\tilde{\theta}}=\frac{1}{\tilde{\theta}^2}\sum_{i=1}^{N}X_i$$

$$\tilde{\theta}=\frac{\sum_{i=1}^{N}X_i}{N}$$