COLORADO STATE UNIVERSITY

Assignment 2 **Agricultural & Resource Economics / Economics 535** *S.R. Koontz*
Fall 2023 **Applied Econometrics**

This assignment is worth 25 points. Please show your work concisely. A printed spreadsheet page is sufficient for calculations. A handwritten document is sufficient for the other questions. Correct answers that are disorganized and do not have clear supporting discussion are worth little. Communication is important. And so is clarity.

1. The following regression equation is a production function where for Q is output, L is labor, and K is capital:

$$\ln(Q_t) = 1.035 + \underset{(0.095)}{0.675} \ln(K_t) + \underset{(0.155)}{0.305} \ln(L_t)$$

$R^2 = 0.395$, $\text{Cov}(\hat{\beta}_K, \hat{\beta}_L) = +0.0004$, $T = 33$, and the standard errors are in parenthesis.

Answer the following questions. When appropriate, be sure to state the null hypothesis, calculate the test statistic, state the critical value, and draw the test conclusion. P-values are also very good to report.

a. Give a brief economic interpretation of each slope coefficient.
b. Set up the appropriate hypotheses and perform the tests to determine if each parameter estimate is significantly different from zero.
c. Set up the appropriate hypothesis and perform the test to determine if the parameter estimates associated with labor and capital are equal.
d. Test the null hypothesis that there are constant returns to scale.
e. You conducted small sample tests for parts b-d. Conduct large sample tests where the sample size is large enough to not matter. (Hint: same statistic but different p-value.)
f. Test the null hypothesis that the regression is insignificant.

2. Consider the following regression model: $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \varepsilon_t$

with sample data:

$n = 20$ $\Sigma X_1 = 350.1$ $\Sigma X_2 = 210$ $\Sigma Y = 559.94$
$\Sigma X_1^2 = 6446.89$ $\Sigma X_2^2 = 2870$ $\Sigma Y^2 = 16057.91$
$\Sigma X_1 X_2 = 3457.7$ $\Sigma X_1 Y = 9492.559$ $\Sigma X_2 Y = 6167.34$.

Refer to Gujarati & Porter Appendix C and especially C.10. Please be sure to set up the matrices and show your work using these. You will need to invert a 3×3 matrix. You can do this by hand, refer to Appendix B, or use software. (Hint: MINVERSE, highlight the cells, F2, ctrl-shift-enter.) Also, correcting for deviations around the mean are only needed for TSS and ESS.

When appropriate, be sure to state the null hypothesis, calculate the test statistic, state the critical value, and draw the test conclusion. Also provide a p-value.

a. Compute the OLS estimates of β_0, β_1, and β_2.
b. Compute the variance-covariance matrix of regression coefficients.
c. Compute R^2 and $\overline{R}^2$.
d. Test the null hypothesis H_0: $\beta_1 = -1$.
e. Test the null hypothesis H_0: $\beta_1 = \beta_2 = 0$.
f. Test the null hypothesis H_0: $\beta_1 = -3\beta_2$.

3. Using the sample from the question above, consider two alternative models:

$$Y_t = \beta_{0S1} + \beta_{1S} X_{1t} + \varepsilon_t \quad \text{and} \quad Y_t = \beta_{0S2} + \beta_{2S} X_{2t} + \varepsilon_t$$

The original model is referred to as the long regression and the two alternative models are short regressions for the obvious reason. Please invert the 2×2 matrices by hand using the methods in Appendix B.

a. Compute the OLS estimates of β_{0S1}, β_{1S}, β_{0S2}, and β_{2S}.
b. Are the estimates of the two slope coefficients different between the long and the two short models? Why?

4. A model was estimated using OLS regression with quarterly data from 1990 through 2022 inclusive. There are four independent variables (k=4). The residual sum of squares was 38.5, and the explained sum of squares was 96.0. Answer the following questions.

a. When three seasonal dummy variables were added to the equation and the equation was re-estimated then the explained sum of squares increased to 99.5. Test for the presence of seasonality.
b. Two further regressions, using the original specification, were estimated for subsamples of 1990 through 2007 and 2008 through 2022. The resulting residual sums of squares were 19.5 and 16.5. First, test if the error variances are equal in the two subsamples. Second, test if the coefficients are identical in the two subsamples.

5. Optional: Determine the Eigenvalues and Eigenvectors of $(X'X)$ and $(X'X)^{-1}$ from the sample used in question 2.