COLORADO STATE UNIVERSITY

Assignment 3 **Agricultural & Resource Economics / Economics 535** *S.R. Koontz*
Fall 2023 **Applied Econometrics**

This problem set is worth 25 points. Please show your work. A hand-written document is sufficient for many questions. Spreadsheet figures would also be sufficient. Correct answers with little supporting discussion are worth little. Communication is important. And so is efficiency.

1. Consider the following model

$$y_t = \alpha + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \beta_4 x_{t-4} + \beta_5 x_{t-5} + \varepsilon_t$$

Show how to derive the estimable model using the polynomial distributed lag model and a second-degree polynomial if (a) there are no end point restrictions and (b) there is a back endpoint restriction ($\beta_6 = 0$). Write the polynomial restrictions where each β_i is a function of ωs. Substitute the functions into the model above for the β's and express the model as linear in the ω_i coefficients. Rename the combinations of the x's as z's. You are deriving an estimable model.

(a) Need an estimable model

$$y_t = \alpha + \omega_0 z_{0t} + \omega_1 z_{1t} + \omega_2 z_{2t} + \varepsilon_t$$

where	could also be
$\beta_1 = \omega_0 + \omega_1 + \omega_2$	$\beta_1 = \omega_1 + \omega_2 + \omega_3$
$\beta_2 = \omega_0 + 2\omega_1 + 4\omega_2$	$\beta_2 = \omega_1 + 2\omega_2 + 4\omega_3$
$\beta_3 = \omega_0 + 3\omega_1 + 9\omega_2$	$\beta_3 = \omega_1 + 3\omega_2 + 9\omega_3$
$\beta_4 = \omega_0 + 4\omega_1 + 16\omega_2$	
$\beta_5 = \omega_0 + 5\omega_1 + 25\omega_2$	as long as there are just three ω's

and
$z_{0t} = (x_{t-1} + x_{t-2} + x_{t-3} + x_{t-4} + x_{t-5})$
$z_{1t} = (x_{t-1} + 2\, x_{t-2} + 3\, x_{t-3} + 4\, x_{t-4} + 5\, x_{t-5})$
$z_{2t} = (x_{t-1} + 4\, x_{t-2} + 9\, x_{t-3} + 16\, x_{t-4} + 25\, x_{t-5})$

(b) Incorporate back endpoint restriction $\beta_6 = \omega_0 + 6\omega_1 + 36\omega_2 = 0$. Solve for one parameter and remove it from the model above and produce an estimable model.

$\omega_0 + 6\omega_1 + 36\omega_2 = 0$ so $\omega_0 = -6\omega_1 - 36\omega_2$

$y_t = \alpha + (-6\omega_1 - 36\omega_2)z_{0t} + \omega_1 z_{1t} + \omega_2 z_{2t} + \varepsilon_t$
$y_t = \alpha + \omega_1 (z_{1t} - 6z_{0t}) + \omega_2 (z_{2t} - 36z_{0t}) + \varepsilon_t$

How many slope parameters in the estimable model? Two.

2. Below are the results from your estimable model (a) above.

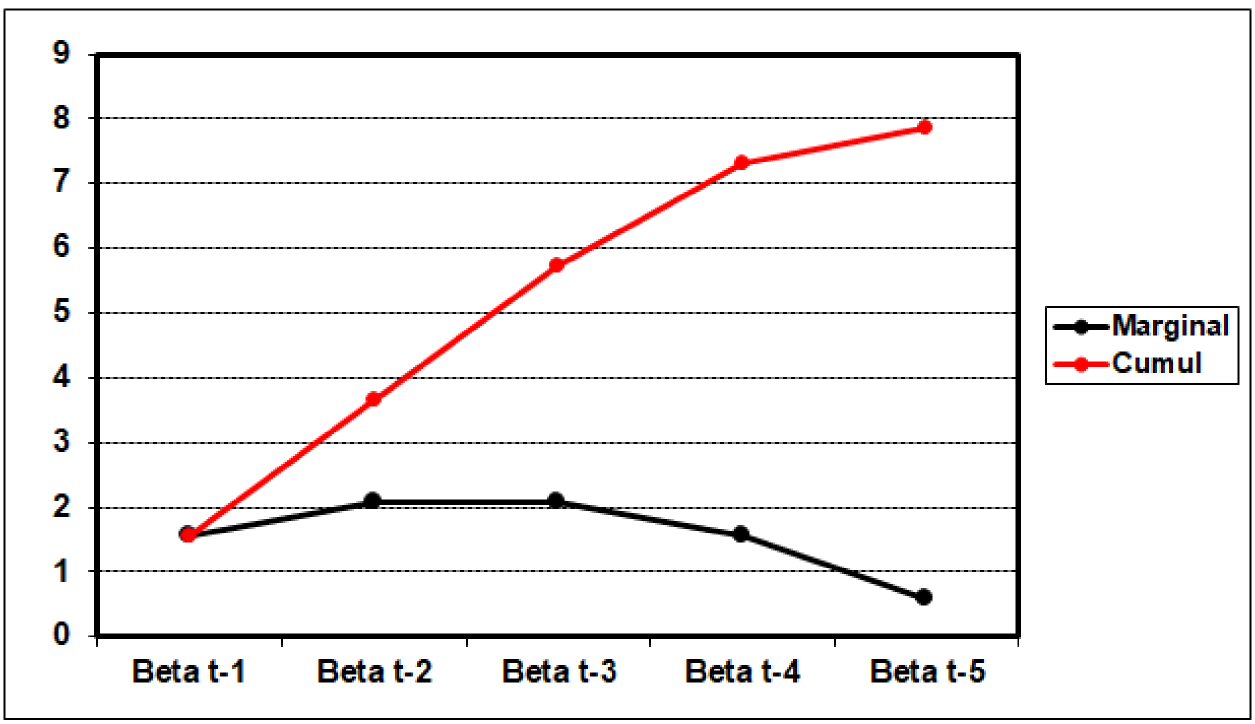
Variables	Coefficient Estimates	Standard Errors	P-Values
Z_0	0.575	0.095	0.0001
Z_1	1.250	0.300	0.0001
Z_2	-0.250	0.195	0.2072

The R^2 for the model was 0.85, the F-Statistic was 47.5, and there was no autocorrelation.

Construct the β's.

	Marginal	Cumul
Beta t-0		
Beta t-1	1.5750	1.5750
Beta t-2	2.0750	3.6500
Beta t-3	2.0750	5.7250
Beta t-4	1.5750	7.3000
Beta t-5	0.5750	7.8750

Must offer precise interpretation of both. Marginal must recognize timing and cet par condition. Cumulative must recognize the additive impact across time. Again, the timing of impacts and cumulations must be precise.



How do you construct standard errors for each β_i? (You do not have sufficient information to do this but you need to communicate the process.) Test if the data finds that your second order polynomial restriction too binding. Interpret the impact multipliers and construct the cumulative impact multipliers. How do you construct standard errors for the cumulative impact measures?

Use the four parameter restriction equation to construct βs. And

$V(\hat{\beta}_1) = V(\hat{\omega}_0) + V(\hat{\omega}_1) + V(\hat{\omega}_2) + 2Cov(\hat{\omega}_0, \hat{\omega}_1) + 2Cov(\hat{\omega}_0, \hat{\omega}_2) + 2Cov(\hat{\omega}_1, \hat{\omega}_2)$

$V(\hat{\beta}_2) = V(\hat{\omega}_0) + 2^2 V(\hat{\omega}_1) + 4^2 V(\hat{\omega}_2)$

$\quad + 2(2)Cov(\hat{\omega}_0, \hat{\omega}_1) + 2(4)Cov(\hat{\omega}_0, \hat{\omega}_2) + 2(2)(4)Cov(\hat{\omega}_1, \hat{\omega}_2)$

…

H_0: $\hat{\omega}_2 = 0$ tests if the second order polynomial is too binding. It is not.

β's are impact multipliers or measures. Σβ are cumulative multipliers. Identification of time period is important.

√V(Σβ) are doable but messy…

3. Consider the following partial adjustment model

$$y_t^* = \alpha + \beta x_t + \varepsilon_t$$

where $y_t - y_{t-1} = \delta(y_t^* - y_{t-1})$ and $0 < \delta \leq 1$.

Derive the estimable model.

Suppose you estimate the following model

$$\underset{}{y_t =} \underset{(0.475)}{1.100} + \underset{(0.125)}{0.625}\, x_t + \underset{(0.075)}{0.725}\, y_{t-1}$$

where standard errors are in parenthesis. The R^2 for the model was 0.945, the F-Statistic was 53.5, and there was no autocorrelation. Construct estimates of the parameters of the original structural model and interpret the parameters. What about standard errors on the structural parameters? (This is a trick question.) Interpret the marginal impacts through time and construct the cumulative impacts. Graph them through 10 periods.

Derive estimable model. Substitute model into adjustment equation

$y_t = \delta\, [\alpha + \beta\, x_t + e_t\,] + (1 - \delta)y_{t-1}$

$y_t = \delta\, \alpha + \delta\, \beta\, x_t + (1 - \delta\,)y_{t-1} + \delta\, e_t$.

$y_t = \beta_0 + \beta_1\, x_t + \beta_2\, y_{t-1} + u_t$

Next derive parameter estimates for structural model from estimable model

$\beta_2 = (1 - \delta)$ so that $\delta = 1 - \beta_2$ or $\delta = 1 - 0.725 = 0.275$.

$\beta_1 = \delta\beta$ so that $\beta = \beta_1/\delta$ or $\beta = 0.625 / 0.275 = 2.2727$.

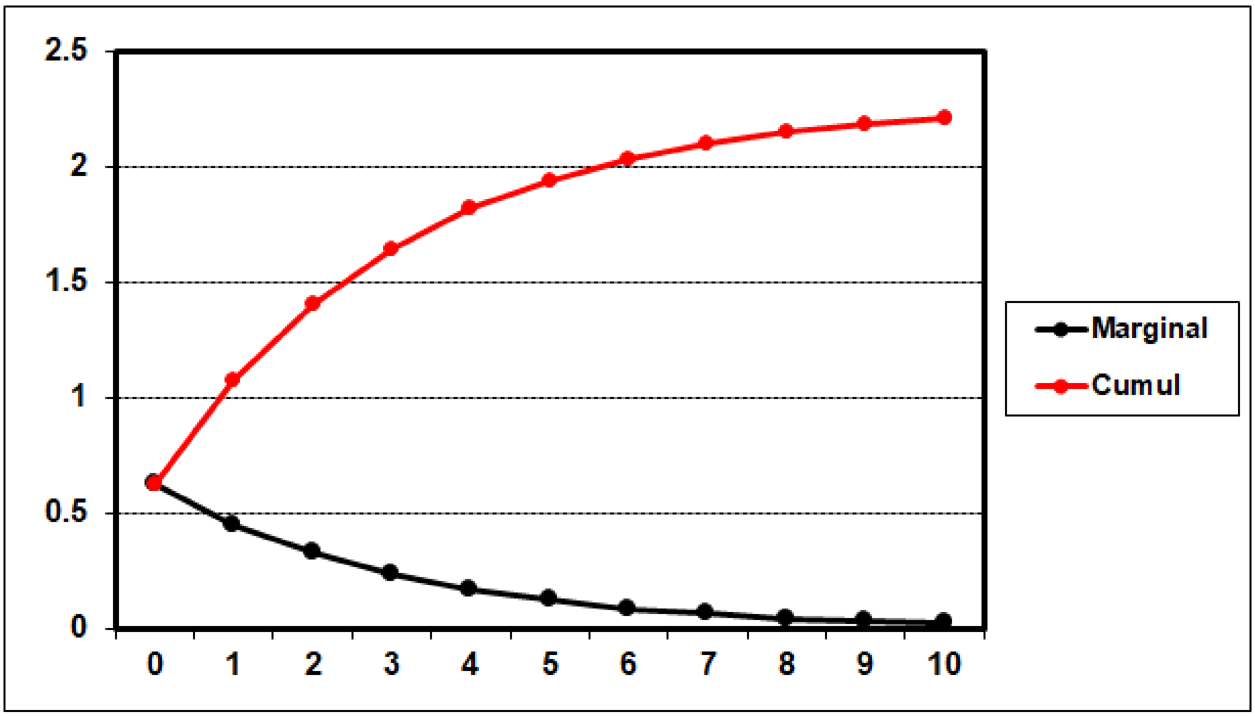
and $\beta_0 = \delta\alpha$ so that $\alpha = \beta_0/\delta$ or $\alpha = 1.100 / 0.275 = 4.0000$.

Interpretation: δ from the expectations model, and slope and intercept from estimable model are not parameters from structural model. Must communicate understand the structural model…

No standard errors: $V(XY) \neq V(X)V(Y)$. (Need to use Delta Rule.)

Except for δ where $V(\delta) = V(1 - \beta_2) = 0 + (-1)^2V(\beta_2) = V(\beta_2)$.

		Marginal	Cumul
0	1	0.625	0.625
1	0.625	0.453125	1.078125
2	0.453125	0.328516	1.406641
3	0.328516	0.238174	1.644814
4	0.238174	0.172676	1.81749
5	0.172676	0.12519	1.942681
6	0.12519	0.090763	2.033443
7	0.090763	0.065803	2.099246
8	0.065803	0.047707	2.146954
9	0.047707	0.034588	2.181541
10	0.034588	0.025076	2.206618

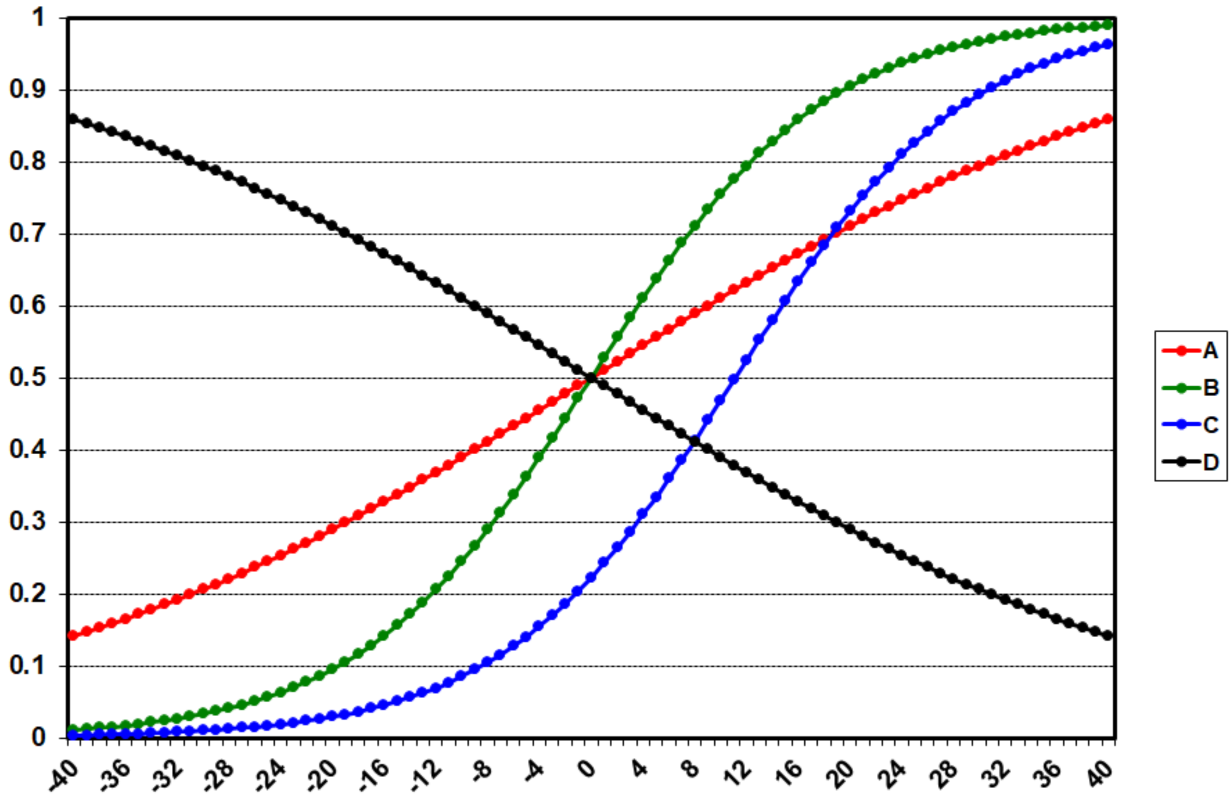


4. Consider the following Logit model with a single explanatory variable

$z_i = \alpha + \beta x_i$

Assume x ranges from -40 to +40. Draw a graph of F(z) for the four pairs of α and β: (0, 0.045), (0, 0.1125), (-1.250, 0.1125), and (0, -0.0450). Increment x by 1 in your figure.

Graphic is as follows:



5. Consider the following Probit model of market choice

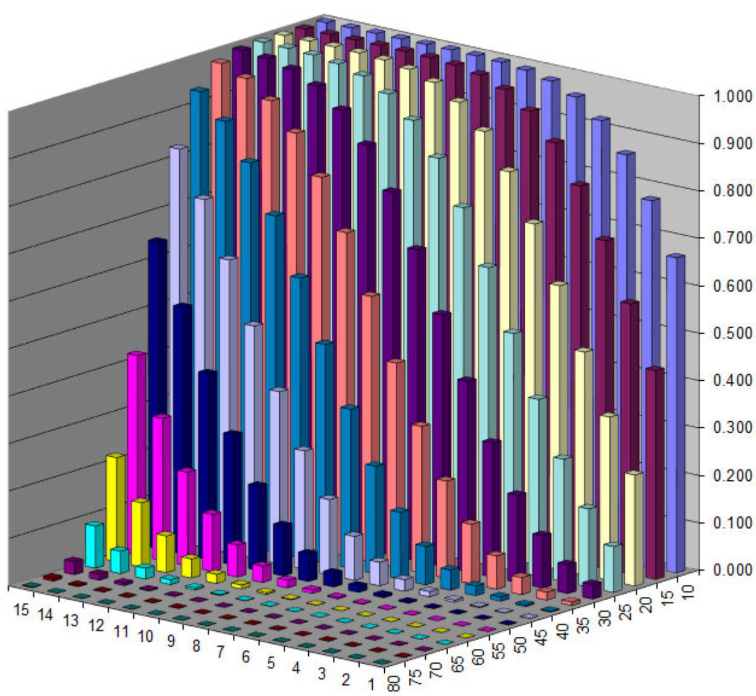
$$z_i = 1.25 + 0.325\, x_{1i} - 0.115\, x_{2i}$$

where x_1 is the percent of income from farming and x_2 is total household income. The dependent variable is whether the farm household markets a farm product through a nontraditional market. The average of $x_1 = 5$ and ranges from 1 to 15. The average of $x_2 = 50$ and ranges from 10 to 80. (The first units are percentages and the second are thousands of dollars. Use these units.)

Educators would like to target outreach efforts at producers with a 50% chance or greater of using nontraditional markets. Can you comment on the attributes that this group of farm households would have?

One equation and two unknowns. Must have information on one x to know about the other. And there is a tradeoff between the two – one variable is positively related to use and the other is negatively related so…

X2 \ X1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10	0.665	0.773	0.859	0.919	0.958	0.980	0.991	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000
15	0.440	0.569	0.691	0.795	0.875	0.930	0.964	0.983	0.993	0.997	0.999	1.000	1.000	1.000	1.000
20	0.234	0.345	0.470	0.599	0.717	0.816	0.890	0.939	0.970	0.986	0.994	0.998	0.999	1.000	1.000
25	0.097	0.165	0.258	0.373	0.500	0.627	0.742	0.835	0.903	0.948	0.974	0.989	0.995	0.998	0.999
30	0.030	0.061	0.110	0.184	0.283	0.401	0.530	0.655	0.766	0.853	0.915	0.955	0.979	0.991	0.996
35	0.007	0.017	0.036	0.070	0.125	0.205	0.309	0.431	0.560	0.683	0.788	0.870	0.926	0.962	0.982
40	0.001	0.003	0.009	0.020	0.042	0.081	0.141	0.227	0.335	0.460	0.589	0.709	0.809	0.885	0.936
45	0.000	0.001	0.002	0.004	0.011	0.024	0.049	0.093	0.159	0.250	0.363	0.490	0.618	0.734	0.829
50	0.000	0.000	0.000	0.001	0.002	0.005	0.013	0.029	0.058	0.106	0.177	0.274	0.392	0.520	0.646
55	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.007	0.016	0.034	0.067	0.120	0.198	0.300	0.421
60	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.019	0.040	0.077	0.136	0.219
65	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.010	0.023	0.047	0.089
70	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.012	0.027
75	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.006



6. Consider the following Tobit model

$y_i = -0.175 + 0.375\ x_i$

where y_i is the dependent variable whereby a large portion of the responses are zero and x_i is the independent variable with mean 0.75. The estimate of $\sigma = 0.185$. (Remember that the betas for the yes/no decision equal β/σ or $z = x\beta/\sigma$ because σ was restricted to one in the qualitative choice model.) Calculate and interpret $\partial E(y)/\partial x$, $\partial F(z)/\partial x$, and $\partial E(y^*)/\partial x$. Calculate and interpret $E(y)$, $F(z)$, and $E(y^*)$.

$x\beta = (-0.175 + 0.375\ (0.75)) = +0.1063$

$z = (-0.175 + 0.375\ (0.75))/0.185 = +0.5743$

$f(z) = 0.3383$

$E(y) = x\beta + \sigma\ f(z)/F(z) = +0.1063 + 0.185\ (0.3383/0.7171) = 0.1935$
Expected value of y given y>0.

$F(z) = 0.7171$
Probability of y>0.

$E(y^*) = x\beta\ F(z) + \sigma\ f(z) = +0.1063\ (0.7171) + 0.185\ (0.3383) = 0.1388$
Expected value of underlying random variable y.

$\partial E(y)/\partial x = \beta[1 - zf(z)/F(z) - f^2/F^2]$
$= 0.375[1 - 0.5743(0.3383)/0.7171 - 0.3383^2/0.7171^2] = 0.1900$
Change in y given y>0 due to a one-unit change in x.

$\partial F(z)/\partial x = f(z)\ \beta/\sigma = 0.3383\ (0.375/0.185) = 0.6857$
Change in probability of y>0 due to a one-unit change in x.

$\partial E(y^*)/\partial x = F(z)(\partial E(y)/\partial x) + (\partial F(z)/\partial x)E(y) = 0.7171(0.1900) + (0.6857)0.1953 = 0.2689$
Change in the underlying random variable y due to a one-unit change in x.

Clear interpretation must be offered.