COLORADO STATE UNIVERSITY

Assignment 4 **Agricultural & Resource Economics / Economics 535** *S.R. Koontz*
Fall 2023 **Applied Econometrics**

This problem set is worth 25 points. It is optional. Please show your work. A hand-written document is sufficient for many questions. Correct answers with little supporting discussion are worth little. Communication is important. And so is efficiency.

A. Consider the following econometric structural model

$$y_{1t} = \beta_{12}\, y_{2t} + \gamma_{11}\, x_{1t} + \gamma_{14}\, x_{4t} + u_{1t}$$

$$y_{2t} = \beta_{21}\, y_{1t} + \gamma_{22}\, x_{2t} + \gamma_{23}\, x_{3t} + u_{2t}$$

with the following

$E(u_{1t}) = E(u_{2t}) = 0$ for all t
$E(u_{1t}\, u_{2s}) = 0$ for all $t \neq s$
$E(u_{it}\, u_{it}) = \sigma_{ii}$ for all t and i = 1,2
$E(u_{it}\, u_{jt}) = \sigma_{ij}$ for all t
x_{1t}, x_{2t}, x_{3t}, and x_{4t} are exogenous and intercepts are assumed, for simplicity, to be zero.

1. Derive the reduced form for this structural model. What is the explicit form of the reduced form disturbances and the reduced form covariance matrix for the disturbances?

2. Determine the necessary (i.e., order) condition for identification of each structural equation. What is the degree of identification? Determine the sufficient (i.e., rank) condition for identification of each equation. Is each equation sufficiently identified?

3. Given the following matrix of sample data in the form of sums of products and cross products

$$\begin{array}{c} y_1 \\ y_2 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{bmatrix} 100 & 200 & 30 & 20 & 40 & 10 \\ 200 & 900 & 0 & 50 & 160 & 100 \\ 30 & 0 & 100 & 0 & 0 & 0 \\ 20 & 50 & 0 & 50 & 0 & 0 \\ 40 & 160 & 0 & 0 & 40 & 0 \\ 10 & 100 & 0 & 0 & 0 & 60 \end{bmatrix}$$

then compute the following. (Caution: the X'X matrix is not realistic as it does not contain an intercept. This is not intended to puzzle you but rather it makes the inversion of X'X very easy in this too-simple-to-be-real example.)

a. The OLS estimates of β_{12}, γ_{11}, and γ_{14} in the first equation.

b. If the OLS estimate for $\sigma_{11} = 1.1$, then compute the OLS estimates of variances for the first equation.

c. The 2SLS estimates of β_{12}, γ_{11}, and γ_{14} in the first equation.

d. If the 2sLS estimate for $\sigma_{11} = 1.4$, then compute the 2SLS estimates of variances for the first equation.

B. The CHICKENS and EGGS data, which have been updated, from Thurman & Fisher (AJAE 1988) are available on the class website. Using only the CHICKENS or the EGGS data series, please use software to

4. Test the series for stationarity. What are the results? If it is nonstationary then first difference the series and conduct all following work with the first-differenced series.

5. Graph the series in levels and first differences.

6. Produce an autocorrelation function (ACF) for the stationary series. Does the function dampen or cutoff? Which lags appear statistically significant?

7. Produce a partial autocorrelation function (PACF) for the stationary series. Does the function dampen or cutoff? Which lags appear statistically significant?

8. Estimate two candidate time series models of the series. Examine the residuals to see if they are white noise. If not, describe. Report your two candidate models. Which lags remain significant? Which model is likely the best?