1. Construct data plots.

2. Summary statistics and correlation coefficients.

Name	N	Mean	Std Dev	Min	Max
Stocks	49	42.5777	51.5343	0.9550	212.30
Bonds	49	1236.94	1045.02	96.58	3509.71
Gold	49	683.013	512.419	134.50	1887.60
CPI	49	69.1731	27.7835	20.80	123.47

ln()	Stocks	Bonds	Gold	CPI
Stocks	1.0000	0.9903	0.7729	0.9807
Bonds		1.0000	0.7817	0.9801
Gold			1.0000	0.8006
CPI				1.0000

3. Three models.

$\ln(\text{Stock Index}_t) = -10.4669 + 3.2205 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.3900)	(0.0936)
(t-stat)	(26.84)	(34.40)
(p-value)	(<.0001)	(<.0001)

R^2 = 0.9618, F-test = 1183.60, P-Value = <.0001, and σ = 0.3063

$\ln(\text{Bond Index}_t) = -3.0709 + 2.3421 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.2886)	(0.0693)
(t-stat)	(10.64)	(33.81)
(p-value)	(<.0001)	(<.0001)

R^2 = 0.9605, F-test = 1143.17, P-Value = <.0001, and σ = 0.2267

$\ln(\text{Gold Price}_t) = 1.1963 + 1.2250 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.5571)	(0.1337)
(t-stat)	(2.15)	(9.16)
(p-value)	(0.0369)	(<.0001)

R^2 = 0.6410, F-test = 83.93, P-Value = <.0001, and σ = 0.4376

4. On average, a one percent change in the CPI index results in (or is associated with) a +3.2205 percent change in the Stock Index. Be exact about units – percent and percent.

   On average, a one percent change in the CPI index results in (or is associated with) a +2.3421 percent change in the Bond Index. Be exact about units.

   On average, a one percent change in the CPI index results in (or is associated with) a +1.2250 percent change in the Gold Price. Be exact about units.

5. $\ln(\text{Stock Index}_{2023}) = -10.4669 + 3.2205 \ln(130.0)) = 5.2090$
   182.9101 points.

   $\ln(\text{Bond Index}_{2023}) = -3.0709 + 2.3421 \ln(130.0)) = 8.3294$
   4143.733 points.

   $\ln(\text{Gold Price}_{2023}) = 1.1963 + 1.2250 \ln(130.0)) = 7.1590$
   1285.663 dollars.

Units?

6. Null Hypothesis: $\beta_1 = 0$.

   Test statistic distributed t with df = 47 and 5%/2 CV = 2.0117.

   <u>Stock Index</u>

   β_1: t = +3.2205/0.0936 = +34.40. Significant at the <.01% level.

   Reject null hypothesis. The probability that the true value of β is insignificantly different zero is very small. The coefficient and regression are significant.

   <u>Bond Index</u>

   β_1: t = +2.3421/0.0693 = +33.81. Significant at the <.01% level.

   Reject null hypothesis. The probability that the true value of β is insignificantly different zero is very small. The coefficient and regression are significant.

   <u>Gold Price</u>

   β_1: t = +1.2250/0.1337 = +9.16. Significant at the <.01% level.

   Reject null hypothesis. The probability that the true value of β is insignificantly different zero is very small. The coefficient and regression are significant.

7. Null Hypothesis: $\beta_1 = 1$.

Test statistic distributed t with df = 47 and 5%/2 CV = 2.0117.

Stock Index

β_1: t = (+3.205 – 1)/0.0936 = 23.7207. Significant at the <.01% level.

F(1,47) = 562.67 Significant at the <.01% level.

Reject null hypothesis of β_1=1.

Bond Index

β_1: (+2.3420 – 1)/0.0693= 19.3747. Significant at the <.01% level.

F(1,47) = 375.38 Significant at the <.01% level.

Reject null hypothesis of β_1=1.

Gold Price

β_1: (+1.2250 – 1)/0.1337 = 1.6826. Significant at the 9.91% level.

F(1,47) = 2.83 Significant at the 9.91% level.

Fail to reject null hypothesis of β_1=1 at standard significance levels.

8. Stocks: slope coefficient is significant, correct sign, and largest F-statistic and R-square.

Bond Index: slope coefficient is significant, correct sign, and very close to the largest F-statistic and R-square.

Gold Price: Slope coefficient is significant, correct sign, and model significant but the least significant.

Argument?: slope coefficient, slope standard errors, residual standard error, F-statistics, and model R-square. Must use facts. Careful with units.

9. Three models.

$\ln(\text{Stock Index}_t) = 2.8655 + 32.2046 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.0438)	(0.9361)
(t-stat)	(65.48)	(34.40)
(p-value)	(<.0001)	(<.0001)

$R^2 = 0.9618$, F-test = 1183.60, P-Value = <.0001, and $\sigma = 0.3063$

$\ln(\text{Bond Index}_t) = 6.6252 + 23.4210 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.0329)	(0.7118)
(t-stat)	204.59)	(33.81)
(p-value)	(<.0001)	(<.0001)

$R^2 = 0.9605$, F-test = 1143.17, P-Value = <.0001, and $\sigma = 0.2267$

$\ln(\text{Gold Price}_t) = 6.2140 + 12.41 \ln(\text{CPI}_t) + e_t$

(Std err)	(0.0621)	(1.3443)
(t-stat)	100.27)	(9.16)
(p-value)	(0.0369)	(<.0001)

$R^2 = 0.6410$, F-test = 83.93, P-Value = <.0001, and $\sigma = 0.4376$

Slopes are the same, but units change by 10 due to re-scaling of CPI.

Must notice the intercepts are different and <u>interpret</u>. Intercepts are means of dependent variables because the X variable is centered on its mean. Correlations and regressions are invariant to linear transformations.

All other sample statistics are identical. So, there’s one thing different and figure out why.

Models are not fundamentally different. OLS models are invariant to linear re-scaling of the data.

10. Summary statistics and correlation coefficients.

Name	N	Mean	Std Dev	Min	Max
%ΔStocks	48	0.1116	0.1253	-0.2203	0.3577
%ΔBonds	48	0.0723	0.0687	-0.1215	0.2464
%ΔGold	48	0.0474	0.1986	-0.3945	0.8178
%ΔCPI	48	0.0371	0.0267	-0.0036	0.1271

	%ΔStocks	%ΔBonds	%ΔGold	%ΔCPI
%ΔStocks	1.0000	0.2516	-0.1079	0.0771
%ΔBonds		1.0000	-0.0115	-0.2969
%ΔGold			1.0000	0.0756
%ΔCPI				1.0000

11. Three models.

$$\%\Delta \text{Stock Index}_t = 0.0981 + 0.3619\ \%\Delta \text{CPI}_t + e_t$$

(Std err)	(0.0314)	(0.6900)
(t-stat)	(3.12)	(0.52)
(p-value)	(0.0031)	(0.6024)

$R^2 = 0.00598$, F-test = 0.28, P-Value = 0.6024, and $\sigma = 0.1262$

$$\%\Delta \text{Bond Index}_t = 0.1007 - 0.7641\ \%\Delta \text{CPI}_t + e_t$$

(Std err)	(0.0165)	(0.3623)
(t-stat)	(6.10)	(2.11)
(p-value)	(<.0001)	(0.0404)

$R^2 = 0.0881$, F-test = 4.45, P-Value = 0.0404, and $\sigma = 0.0663$

$$\%\Delta \text{Gold Price}_t = 0.0265 + 0.5624\ \%\Delta \text{CPI}_t + e_t$$

(Std err)	(0.0443)	(1.0941)
(t-stat)	(0.42)	(0.51)
(p-value)	(0.6764)	(0.6097)

$R^2 = 0.0057$, F-test = 0.26, P-Value = 0.6097, and $\sigma = 0.2002$

12. On average, a one percent year-on-year change in the CPI index results in a +0.3619 percent year-on-year change in the Stock Index. Insignificant. Be exact about units.

On average, a one percent year-on-year change in the CPI index results in a -0.7641 percent year-on-year change in the Bond Index. Insignificant. Be exact about units.

On average, a one percent year-on-year change in the CPI index results in a +0.5624 percent annual change in the Gold Price. In/Significant? Be exact about units.

13. Null Hypothesis: $\beta_0 = 0$ and $\beta_1 = 1$.

Test statistic distributed F(2,46) and CV = 3.1996.

Stock Index

F(2,46) = 8.78 Significant at the 0.06% level.

$\chi^2(2)$ = ? Significant at the 0.06% level.

Reject null hypothesis of asset price following inflation.

Bond Index

F(2,46) = 18.63 Significant at the 0.01% level.

$\chi^2(2)$ = ? Significant at the 0.01% level.

Reject null hypothesis.

Gold Price

F(2,46) = 0.14 Significant at the 86.68% level.

$\chi^2(2)$ = ? Significant at the 86.68% level.

Fail to reject null hypothesis.

14. Stocks: slope coefficient is insignificant, but correct sign, and the model is least significant. Not related to inflation.

Bonds. Slope coefficient is insignificant, incorrect sign, and intermediate in terms of significance. Opposite with inflation.

Gold Price: Slope coefficient is almost significant, correct sign, and largest F-statistic and R-squared. Same as inflation.

Data in levels are long run. Data in changes are short run. The data and the model have been fundamentally changed.

Recommend: Stocks in the long run: better than. Bonds okay in long-run just not as good as stocks – as evidenced by the two slope estimates. Gold in the short run: same as.

15. Optional

$\%\Delta Stock\ Index_t = 0.0871 + 0.6438\ \%\Delta CPI_t - 0.1018\ (Y_{t-1} - X_{t-1}) + e_t$

(Std err)	(0.0316)	(0.6982)	(0.0613)
(t-stat)	(2.76)	(0.92)	(1.66)
(p-value)	(0.0083)	(0.3614)	(0.1040)

$R^2 = 0.0633$, F-test = 1.52, P-Value = 0.2298, and $\sigma = 0.1239$

$\%\Delta Bond\ Index_t = 0.0965 - 0.6398\ \%\Delta CPI_t - 0.1222\ (Y_{t-1} - X_{t-1}) + e_t$

(Std err)	(0.0152)	(0.3354)	(0.0397)
(t-stat)	(6.34)	(1.91)	(3.08)
(p-value)	(<.0001)	(0.0628)	(0.0036)

$R^2 = 0.2467$, F-test = 7.37, P-Value = 0.0017, and $\sigma = 0.0609$

$\%\Delta Gold\ Price_t = 0.0036 + 1.1589\ \%\Delta CPI_t - 0.0998\ (Y_{t-1} - X_{t-1}) + e_t$

(Std err)	(0.0520)	(1.1648)	(0.0727)
(t-stat)	(0.07)	(0.99)	(0.99)
(p-value)	(0.9459)	(0.3251)	(0.1708)

$R^2 = 0.0468$, F-test = 1.10, P-Value = 0.3405, and $\sigma = 0.1982$

The short-run variable measures how short-run changes in x relate to short-run changes in y. And the new measure is a long-run measure. Both in the same model.

The negative sign on the new long-run variable implies as the error gets bigger (positive or negative) then this long-run coefficient pulls the two series back together.

Stocks move with inflation in the short run (but not significantly) and with inflation in the long run (but not significantly). Bonds move opposite with inflation in the short run (but almost significantly) and with inflation in the long run (significantly). Gold moves with inflation in the short run (but not significantly) and with inflation in the long run (but not significantly).

Short-run and long-run conclusions with this model?

I think stock returns just grow faster than inflation. Likewise, bond returns but just slower than stocks. Gold is the same as inflation over the very long term.