Panel Data

From the perspective of an applied micro economist

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Slides partially taken from Stephen Koontz

What is panel data?

What is panel data?

Generally, a mixture of cross-sectional and time series data

$y_{it} = \beta_0 + \beta_1 x_{1it} + \beta_2 x_{2it} + \ldots + \beta_k x_{kit} + e_{it}$

where $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Sample size is $N \times T$. This is a Balanced Design.

Example of an Unbalanced Design: $i = 1, \ldots, N_t$ and $t = 1, \ldots, T_i$. Each i has a different number of T. Or...?

What do the data matrices look like?

$$y = \begin{bmatrix} y_{11} \\ . \\ y_{1T} \\ y_{21} \\ . \\ y_{2T} \\ . \\ . \\ y_{N1} \\ . \\ y_{NT} \end{bmatrix} \qquad X = \begin{bmatrix} 1 & x_{1\,11} & \ldots \\ . & . & \\ 1 & x_{1\,1T} & \\ 1 & x_{1\,21} & \\ . & . & \\ 1 & x_{1\,2T} & \\ . & . & \\ . & . & \\ 1 & x_{1\,N1} & \\ . & . & \\ 1 & x_{1\,NT} & \end{bmatrix}$$

What are some examples of panel data?

Balanced vs. unbalanced panels

- What are they?
- When is an unbalanced panel a problem?

Why might we prefer panel data?

- We can exploit variation within an individual (i) over time
- We can exploit variation within time periods across individuals

- But why might this help us as econometricians?

We are interested in…

- Establishing an argument for causality: x causes a β change in Y

$$Y_{it} = \alpha + \beta_1 x_{it} + \epsilon_{it}$$

- What are the key threats to this argument?
    - This is called identification

Bias and omitted variables

- What is omitted from this equation that could lead to biased estimates of β_1?

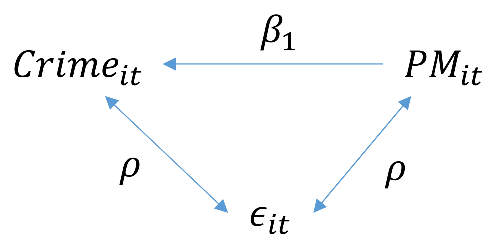
$$Y_{it} = \alpha + \beta_1 x_{it} + \epsilon_{it}$$

Example: Pollution and Crime

$$Crime_{it} = \alpha + \beta_1 PM_{it} + \epsilon_{it}$$

- $Crime_{it}$= crime in county i during time t
- PM_{it}=particulate matter in county i during time t

- What is omitted from this equation that could lead to biased estimates of β_1?



Omitted unobservables

- Let's consider two categories of unobservables
  - Things that are county constant but vary over time
  - Things that are time constant but vary across counties

- How might we control for these unobservables?
- Hint: how do we control for gender?

Two options (for today)

- Fixed effects
- Random effects

Fixed-Effects Models

Suppose we want each ith individual to have its own mean...

$$y_{it} = \beta_0 + \beta_1 x_{1it} + \beta_2 x_{2it} + \ldots + \beta_k x_{kit} + \sum \alpha_i D_i + e_{it} \qquad e_{it} \sim N(0, \sigma^2)$$

where D_i = 1 for observation on ith individual, and 0 otherwise.

Suppose we want each tth time period to have its own mean...

$$y_{it} = \beta_0 + \beta_1 x_{1it} + \beta_2 x_{2it} + \ldots + \beta_k x_{kit} + \sum \theta_t D_t + e_{it} \qquad e_{it} \sim N(0, \sigma^2)$$

where D_t = 1 for observation on tth period, and 0 otherwise.

Example: Pollution and Crime

$$Crime_{it} = \beta_1 PM_{it} + \sum_i \alpha_i D_i + \sum_t \gamma_t D_t + \epsilon_{it}$$

- $Crime_{it}$= crime in county i during time t
- PM_{it}=particulate matter in county i during time t
- $\alpha_i D_i$= fixed effects for each county
- $\gamma_t D_t$= fixed effects for each time period

- What do these fixed effects control for?
- Are there still omitted variables that could lead to biased estimates of β_1?

Quick Aside: how are fixed implemented?

1. Dummy variables

$$Crime_{it} = \beta_1 PM_{it} + \sum_i \alpha_i D_i + \epsilon_{it}$$

2. Demean by i:

$$(Crime_{it} - \overline{Crime_i}) = \beta_1 (PM_{it} - \overline{PM_i}) + (\epsilon_{it} - \overline{\epsilon_i})$$

3. Equivalent to first differences with 2 time periods

Can throw in time period dummies in either model.
Why are these equivalent?

Fixed Effects Assumptions

For the model $Y_{it} = \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + e_{it}, t = 1, \ldots, T$

1) β_k are the parameters to estimate and a_i is the unobserved effect
2) We have a random sample from the cross sections (unbalanced?)
3) Each x changes over time. **Why?** And no perfect multicollinearity
4) For each t, the expected value of the idiosyncratic error given the explanatory variables in *all* time periods and the unobserved effect is zero: $E(e_{it}|x_{ik}, a_i) = 0$
   - This is the strict exogeneity assumption

- Under these assumptions, the FE estimator is unbiased

Fixed Effects Assumptions Cont.

For the model $Y_{it} = \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + e_{it}, t = 1, \ldots, T$

5) $var(e_{it}|x_i, a_i) = var(e_{it}) = \sigma_e^2$, for all t=1,…,T.

This can be addressed with heteroskedasticity robust standard errors

6) For all $t \neq s$, the idiosyncratic errors are uncorrelated: $cov(e_{it}, e_{is}|x_i, a_i) = 0$

Implies…

Benefits of FE

- Makes no assumptions about the correlation between a_i and x_i

Drawbacks of FE

- Suppose we have the model:

$$Crime_{it} = \beta_1 PM_{it} + \sum_i \alpha_i D_i + \sum_t \gamma_t D_t + \epsilon_{it}$$

We cannot include variables that are constant within counties and we cannot include variables that are constant within a year.

* Examples: whether or not a county is urban, geographic region of the US, national policies that do not vary over time.

Random Effects

- What if we want to estimate parameters of variables that are constant within counties, but still control for county specific unobservables?
- Random effects allow us to do this, with an additional assumption.

RE assumptions

Given the model

$$Y_{it} = \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + e_{it}$$

- Fixed effects allows for correlation between a_i and x's.
- But what if we think a_i and the x's are uncorrelated in all time periods?
  - Example of when this might be the case?
- Thus, the RE assumptions are the same as the fixed effects assumptions with the additional assumption:
  - a_i is independent of all explanatory variables in all time periods: $cov(x_{itj}, a_i) = 0$

How is RE implemented?

$$Y_{it} = \beta_0 + \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + e_{it}$$

Combined error:

$$Y_{it} = \beta_0 + \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + v_{it}$$

where

$$v_{it} = a_i + e_{it}$$

Because a_i is contained in v_{it}, the composite errors are serially correlated, described by

$$corr(v_{it}, v_{is}) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_e^2}, t \neq s$$

Where $\sigma_a^2 = var(a_i)$ and $\sigma_e^2 = var(e_{it})$

To address this use weighted LS with weights defined as follows

$$\lambda = 1 - \left[\frac{\sigma_a^2}{T\sigma_a^2 + \sigma_e^2}\right]^{1/2}$$

Which is between 0 and 1 (this is important)

The transformed RE equation is

$$Y_{it} - \lambda\overline{Y_i} = \beta_0(1-\lambda) + \beta_1(x_{it1} - \lambda\overline{x_{i1}}) + \cdots + \beta_k(x_{k1} - \lambda\overline{x_{k1}}) + (v_{it} - \lambda\overline{v}_i)$$

- The FE estimator subtracts the time averages
- The RE estimator subtracts a fraction of the time averages
- This also solves the serial correlation in v
- Sample analogs are computed from OLS estimates of v
- Pooled OLS is obtained when $\lambda = 0$
- The RE estimator tends towards the FE estimator as λ goes to 1

Drawback of RE

- Need to assume a_i are uncorrelated with x_i in all time periods which is unlikely.

Benefit of RE

- Plausibly controls for time constant individual specific unobservables while allowing for the recovery of parameters on time constant individual specific covariates.

Example

$$Violent\ Crime_{it} = \beta_1 PM_{it} + D_i + D_t + \epsilon_{it}$$

- $Violent\ Crime_{it}$ count in county i on day t
- PM_{it} is a measure of air pollution in county i on day t
- D_i is a location fixed effect or random effect
- D_t is a time fixed effect

Table 8: Violent Crimes RE and FE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$PM_{2.5}$	0.123***	0.112***	0.059***	0.060***	0.012***	0.013***	0.012***	0.012***
	(0.008)	(0.008)	(0.008)	(0.008)	(0.002)	(0.002)	(0.002)	(0.002)
year FE		Y	Y	Y	Y	Y	Y	Y
state			FE	RE				
county					FE	RE	FE	RE
month FE							Y	Y
N	77489	77489	77489	77489	77489	77489	77489	77489

Example 2: What explains wages?

$$Wage_{it} = \beta_0 + \beta_1 educ_i + \beta_2 black_i + \beta_3 hispan_i + \beta_4 exper_i$$
$$+\beta_5 exper_{it}^2 + \beta_6 married_{it} + \beta_7 union_{it} + \phi + e_{it}$$

Which variables will drop out with individual FE?

- Time constant parameters are similar for OLS and RE
- Marriage and union premiums fall from OLS to RE. Why?
- Eliminate the household unobservable entirely using FE, the parameters fall even more (why?)
- Captures the idea that people that are more able (higher a_i) are more likely to be married and more likely to have higher wages.
- In OLS, a large part of marriage coefficient is due to the fact that most people who are married would earn more even if they weren't married.

TABLE 14.2

Three Different Estimators of a Wage Equation

Dependent Variable: log(*wage*)			
Independent Variables	**Pooled OLS**	**Random Effects**	**Fixed Effects**
educ	.091 (.005)	.092 (.011)	——
black	−.139 (.024)	−.139 (.048)	——
hispan	.016 (.021)	.022 (.043)	——
exper	.067 (.014)	.106 (.015)	——
exper^2	−.0024 (.0008)	−.0047 (.0007)	−.0052 (.0007)
married	.108 (.016)	.064 (.017)	.047 (.018)
union	.182 (.017)	.106 (.018)	.080 (.019)

Final thoughts

- There is a test called the Hausman test for Fixed versus Random Effects
- Null hypothesis is that the effects are uncorrelated with the data (x's), or random effects are acceptable.
- Most often will reject in favor of FE and that's why you see FE used in most economics studies.