

# Integrating Long-Term and Short-Term Contracting in Beef Supply Chains

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This paper analyzes the optimal procurement, processing, and production decisions of a meat-processing company (hereafter, a “packer”) in a beef supply chain. The packer processes fed cattle to produce two beef products, program (premium) boxed beef and commodity boxed beef, in fixed proportions, but with downward substitution of the premium product for the commodity product. The packer can source input (fed cattle) from a contract market, where long-term contracts are signed in advance of the required delivery time, and from a spot market on the spot day. Contract prices are taken to be of a general window form, linear in the spot price but capped by upper and lower limits on realized contract price. Our analysis provides managerial insights on the interaction of window contract terms with processing options. We show that the packer benefits from a low correlation between the spot price and product market uncertainties, and this is independent of the form of the window contract. Although the expected revenues from processing increase in spot price variability, the overall impact on profitability depends on the parameters of the window contract. Using a calibration based on the report by the GIPSA (Grain Inspection, Packers and Stockyards Administration. 2007. GIPSA livestock and meat marketing study, vol. 3: Fed cattle and beef industries. Report, U.S. Department of Agriculture, Washington, DC), this paper elucidates for the first time the value of long-term contracting as a complement to spot sourcing in the beef supply chain. Our comparative statics results provide some rules of thumb for the packer for the strategic management of the procurement portfolio. In particular, we show that higher variability (higher spot price variability, product market variability, and correlation) increases the profits of the packer, but decreases the reliance on the contract market relative to the spot market.

*Key words:* contracting; beef supply chain; commodity risk management; multiproduct newsvendor; window contracts

*History:* Received October 26, 2009; accepted March 11, 2011, by Yossi Aviv, operations management.

Published online in *Articles in Advance* August 4, 2011.

## 1. Introduction

The purpose of this paper is to develop a theoretical basis for understanding the trade-offs facing a meat-processing company (hereafter, a “packer”) in the choice of alternative arrangements for sourcing fed cattle, when that packer acts as a wholesaler into several final product markets. This is an example of a broader class of risk management and contracting problems, including petroleum and many agricultural products, in which a single primary input gives rise to multiple outputs. The resulting interdependencies between procurement practices for the primary input and downstream markets present new challenges for supply chain management.

We examine these challenges in the context of the U.S. beef industry, which is the largest single industry

within U.S. agriculture, generating between \$34 and \$37 billion per year in 2006–2008 and accounting for 20% of the annual total market value of agricultural products sold in the United States (U.S. Department of Agriculture 2009). A similar analysis would apply to other cattle-producing regions of the world that rely for fed-cattle procurement on a mix of spot markets and long-term contracts (e.g., Europe and South America). Although this paper will focus on the beef supply chain for specificity, much of our analysis would also apply to other live-animal supply chains such as pork hog, broiler chicken and lamb, and to other supply chains that have a common input from which multiple outputs are produced.

The beef industry is a combination of assembly and disassembly and of product flow smoothing.

The base production unit in the industry—the beef cow herd—lives outdoors and consumes grass-based forage. After obtaining cheap growth of the animal frame, the animals are referred to as “feeder cattle” and are assembled by the cattle-feeding industry. Feeder animals feed for four to six months depending on seasonal factors, such as energy requirements due to living outdoors and seasonal demand for beef consumption, and grain prices relative to beef prices. Finished animals are referred to as “fed cattle” and are marketed to packers.

As reported in the GIPSA (2007) report, there are some 25 large commercial fed-cattle slaughtering and processing facilities in the United States. And it is here that disassembly begins. Each animal can be used to produce a subset of hundreds of standard beef cuts. These are packaged as premium products (program boxed beef) or commodity products (commodity boxed beef). Food service firms such as restaurant chains may procure program beef. Grocery stores market a variety of commodity beef. There are distinct differences in regional and seasonal demand patterns across the United States for different beef products.

Several interlinked markets operate to determine pricing and delivery quantities at various stages along the beef supply chain. We will focus on the two markets of greatest interest to packers (see Figure 1):

1. the market between processors/packers and all upstream elements (including feedlots and prior elements) of the beef value chain;
2. the market between processors/packers and all downstream elements (including wholesalers and retailers) of the beef value chain.

Considering the upstream elements in the beef supply chain, there are actually two markets of interest: the spot market and the contract market.

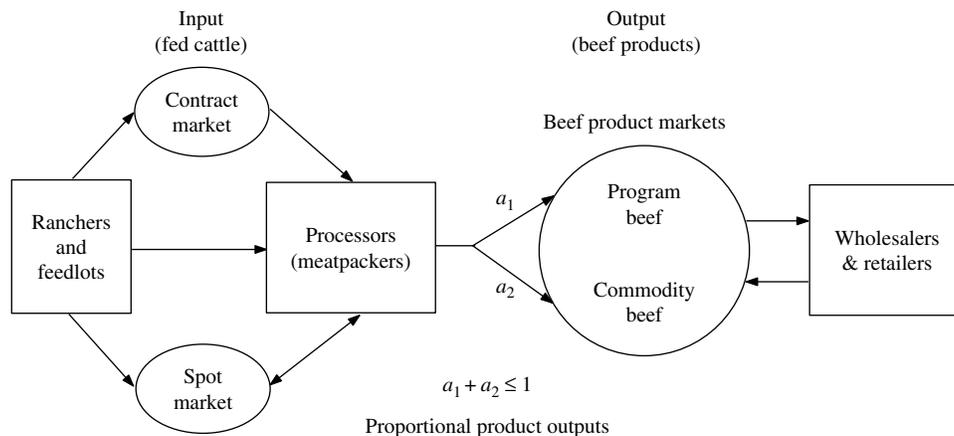
Spot markets (also referred to as cash markets) are real-time regional markets for transactions of fed cattle, often through auctions. In keeping with the extensive literature on the subject, e.g., GIPSA (2007), we

will assume throughout that spot markets are competitive, i.e., the price is not sensitive to the actions of any of the agents (buyers or sellers) who participate in this market.

Contract markets feature longer-term arrangements between feedlot owners and packers. The contracts themselves are often referred to as “marketing agreements.” Such agreements may allow some flexibility in the quantity delivered, in the usual options form, or have more advanced features in pricing of yield risks (grid or formula based) than fixed forwards based on live-weight metrics. We analyze here a general class of “window” contracts, with contract price equal to a linear function of the spot price when the resulting contract price is in a window between fixed upper and lower limits, and otherwise capped by the indicated limits. As also discussed by Li and Kouvelis (1999), window contracts provide a risk-sharing mechanism between the buyer and the seller for spot price risk exposure. This general contract form includes firm fixed forwards as a special case (when the upper and lower limits coincide) as well as the “standard contract,” most common in the industry, which has no upper or lower limits on contract price. The standard contract specifies the price per unit on the basis of the spot price prevailing at a specified market on delivery day, plus a fixed surcharge. The fixed surcharge is intended to cover the cost of additional feeding specifications that are part of the contract and that give rise to the additional value of contract cattle resulting from the higher percentage of premium product (program beef) in these cattle. Contract cattle can also be resold in the spot market by the contracting packer if they are not needed for production.

For packers in the United States, the spot market is a very important source of physical supply, averaging for many packers in excess of 60% of total supply according to the GIPSA (2007). The heavy reliance on the spot market noted in the GIPSA (2007) report is

**Figure 1** Upstream and Downstream Elements for Packers in Beef Supply Chain



driven in part by the large number of small producers of cattle, who raise cattle as complements to their other farming activities, and the fact that spot sales in organized markets are an efficient way of bringing such cattle to market. Contract purchases obtained from larger feedlots offer certain advantages to packers such as the ability to contract for and monitor special feeding regimes that are intended to increase the quality of meat produced.

Focusing on a single packer, we consider the optimal mix of contract and spot purchases in providing input from upstream feedlots and spot markets. Our analysis shows the impact on this portfolio of spot price and demand uncertainty and correlation and the degree of substitution between products in final markets. We assume that neither the cattle nor the finished products can be inventoried—they have a certain “ripe” or sale date toward which all contracting is directed.<sup>1</sup> Because the focus is on the short and medium term, capacity and processing technology are also assumed fixed.

This paper intends to make contributions in two areas: (1) in the analysis of general window contracts common in agricultural and metals supply chains, and (2) in the analysis of a benchmark case for the most important U.S. agricultural market, beef. We undertake both of these analyses for fixed proportion technologies, which entail the production of multiple outputs from a single primary input, with downward product substitution possibilities, which entail the conversion of premium output to standard output.

Our analysis of window contracts on the primary input focuses on the interactions of the contract terms with processing options, including product substitution, and the associated revenues of processed product. We demonstrate (with normally distributed spot price and symmetric window around the forward price) that the value of using a window contract instead of a fixed forward contract and its implications on the optimal procurement portfolio are determined by the ordering between the forward price and the mean contract procurement price. Our comparative statics results provide managerial insights on the interaction of contract terms with processing options. We show that the firm benefits from a lower correlation between the spot price and product market uncertainties. We also show that the expected revenues from processing increase in spot price variability, but the overall impact on profitability depends on the parameters of the window contract. In the

absence of spot procurement, the firm should increase its contract volume with a lower correlation, whereas the same holds with a higher spot price variability if the window contract does not have a lower upside protection than the downside opportunity loss. With spot procurement, the impact of the correlation and the spot price variability on the optimal procurement portfolio is more subtle and is determined by the interplay between the spot price and product market uncertainties.

Our contributions on the beef industry focus on the central player in these markets, the packer. Specializing our generalized contract form to the standard contract in use in the industry, we illustrate the significant impact on profits of integrated risk management in this fixed proportions supply chain. In particular, using a calibration based on the GIPSA (2007) report, this paper elucidates for the first time the value of long-term contracting in the beef supply chain. This has been a point of continuing controversy in the policy debate concerning the structure and operations of the beef industry. Our analysis provides some rules of thumb for the packer. We demonstrate that higher variability (higher spot price variability, product market variability, and correlation) increases the profits, but decreases the value of the contract market relative to the spot market. We also show that higher demand substitution is detrimental to the packer’s profitability and reduces dependence on contract procurement, but product substitution does not have any significant effect on the packer’s decisions and performance.

This paper proceeds as follows. We review relevant literature in the next section. Thereafter follows our model development in §3 and its optimal solution in §4. This model development provides a general solution for window contracts based on payoffs that are linear in an underlying spot market, with comparative statics for this general model provided in §5. Section 6 provides numerical simulations for the GIPSA (2007) data and for the standard contract form used in U.S. beef markets, a special case of our general window contracts. These results also provide comparative statics of model results for product market and spot market parameters of interest. We conclude in §7 with a discussion of limitations of our analysis and the path forward for future research.

## 2. Literature Review

The focus of this paper is on supply chain contracting in the presence of spot markets. See Cachon (2003) and Kleindorfer and Wu (2003) for a review of the literature related to this theme. Assuming a competitive spot market (i.e., based on large numbers of interacting buyers and sellers), Wu and Kleindorfer (2005) provide conditions such that it is optimal for buyers to source from both the contract market and the

<sup>1</sup> Following the GIPSA (2007) report, herds are treated as inventory or investment goods but fed cattle must be marketed within a two- to three-week window or face substantial feeding cost penalties and meat quality penalties. Likewise, fresh beef is sold under the old adage: “sell it or smell it.”

spot market. Mendelson and Tunca (2007) provide an alternative rationale for the existence of simultaneous forward and spot market sourcing, based on strategic spot trading. A similar closed spot market model was used by Chod et al. (2010a). We do not consider such strategic spot market interactions in this paper because (as noted in the GIPSA 2007 report) spot markets for fed cattle, our target application, have large numbers of informed participants, transparent in their function and competitive in their operation.

Two streams of literature are evidently related to the multiple-output character of the beef problem. The first stream of papers analyzes coproduction systems where multiple outputs are produced simultaneously in a single production run (Gerchak et al. 1996, Bitran and Gilbert 1994, Hsu and Bassok 1999, Tomlin and Wang 2008). The standard coproduction problem foresees different grades or quality levels of output, where yields for these different grades are typically random. The problem of contracting for inputs (e.g., wafer starts in semiconductor manufacture) when facing demand schedules for each of the grades has some similarities to the beef-processing problem, including downward substitution in production. However, the primary focus in the coproduction literature is on the production quantity and the allocation of the realized production output to the product demands, whereas the primary focus in a proportional output setting such as beef markets is on integration of upstream and downstream pricing and contracting.

A second stream of papers related to the multiple-output character of the beef problem is the literature on newsvendor network models. As defined by Van Mieghem and Rudi (2002), newsvendor networks encompass the structural properties of the single-product newsvendor problem and extend this to the multiproduct setting (with non-price-sensitive stochastic demand). We refer the reader to Dong et al. (2010a) for a review of papers using a newsvendor network formulation and for further applications in the context of multiple markets and transshipment networks. The beef supply chain context requires a generalization of the newsvendor network model to include pricing and proportional input–output relationships, with substitution possibilities both in production and in demand. These are essential generalizations for beef supply chains and many others (e.g., petroleum and many agricultural products) that involve input–output interdependencies that transcend the supply network configuration and involve the product structure itself.

The literature on supply chain management issues in the agricultural sector has focused mainly on uncertain yields and contracting issues related to multi-factor supply chains. In this regard, Kazaz (2004) analyzes the choice between long-term contracts and a

secondary supply option with yield uncertainty with a special focus on the olive industry. Burer et al. (2008) look at supply chain coordination issues in the seed industry, focusing on different contract types prevalently used in practice. Lowe and Preckel (2004) provide a summary of literature on crop production. The GIPSA (2007) provides an extensive literature review of the beef industry, which is updated and supplemented by Boyabatlı et al. (2011). The essential contribution of the present paper relative to this earlier work is the explicit treatment and integration of fixed proportion output markets with upstream market characteristics and contracting decisions.

Against the background of the above literature, we note several important lacunae. For the upstream market, there is no research on the optimal mix of procurement methods (contract versus spot) within the beef industry. This is an important matter from a policy perspective, as the above discussion of the GIPSA (2007) report and the controversy concerning contract markets make clear. Furthermore, the key issue of quality/yield risks (which differ across contract and spot cattle) needs to be integrated with production and demand management. For the downstream market, the key issue is that of multiple products arising from processing and the demand uncertainties and substitution effects associated with these. It is on these issues, and their related impacts on optimal processing decisions for the packer, that we focus our model and our results. We begin with a general treatment of spot-based window contracts, which we then specialize to the beef supply chain.

### 3. Model Description

#### 3.1. Notation and Preliminaries

A realization of the random variable  $\tilde{\mathbf{y}}$  is denoted by  $\mathbf{y}$ . Bold letters represent vectors of the required size. Vectors are column vectors and the prime ( $\prime$ ) denotes the transpose operator. We have  $(u)^+ = \max(u, 0)$  and  $\Omega^{12} = \Omega^1 \cup \Omega^2$ .  $\Pr$  denotes probability, and  $\mathbb{E}$  denotes the expectation operator. Monotonic relations are used in the weak sense unless otherwise stated. “C-input” denotes the input sourced from the contract market, and “S-input” denotes the input sourced from the spot market. The cumulative distribution function (cdf) and probability density function (pdf) of the standard normal random variable are denoted by  $\Phi(\cdot)$  and  $\phi(\cdot)$ , respectively.

We consider a firm (the packer in the beef setting) that procures and processes a single primary input (fed cattle) to produce two final products, a premium (program beef) and a standard product (commodity beef). We model the firm’s procurement, processing, and production decisions in a two-period framework.

### 3.2. Procurement

We consider two sources for procurement, contracts and spot markets. A typical contract specifies the volume of C-input committed by the firm in advance of the spot market and delivered to the firm on the spot day. The firm can also buy S-input from the spot market on the day. Let  $Q^C$  denote the volume of C-input, and let  $Q^S(P^S)$  denote the volume of S-input at the prevailing spot price  $P^S$ . We assume that  $\tilde{P}^S$  has a continuous distribution with positive support with finite expectation  $\mu_S$  and standard deviation  $\sigma_S$ .

There are differences between C- and S-input in terms of quality, processing cost, and contract price. C-input is priced as a linear function of spot price, capped by upper and lower limits on C-input price.<sup>2</sup> Formally, the unit price of C-input on the day is  $\max[\min(u, P^S + \nu), l]$  with  $l \leq u$ . Here  $l$  is the lower bound,  $u$  is the upper bound on the contract price, and  $\nu$  is a contract-specific per-unit adjustment (to account for differences in quality, delivery terms, and other matters that distinguish C-input from S-input). We note that a pure forward contract is obtained as a special case of the window contract when  $l = u$ .

The unit price of the S-input is the prevailing spot price  $P^S$  with an additive transaction cost  $t \geq 0$  applied. This transaction cost reflects transportation cost from the spot market to the firm's plant. The firm can also resell C-input, which it receives in the spot market at a unit sales price of  $(1 - \omega)P^S$ , where  $0 \leq \omega \leq 1$  represents a discount or transaction cost. We assume  $\mathbb{E}[\max[\min(u, P^S + \nu), l]] > (1 - \omega)\mu_S$  (i.e., expected contract price is higher than expected spot resale revenue per unit), because otherwise C-input would dominate S-input.

### 3.3. Processing

We define  $\mathbf{z}' = (z^C, z^S)$  as the processed input vector composed of C-input,  $z^C$ , and S-input,  $z^S$ . We assume a processing capacity constraint  $K$  (hereafter referred to as plant size) such that  $\mathbf{1}'\mathbf{z} \leq K$ , and the total processing cost is denoted by  $C(\mathbf{z}) = c_0\mathbf{1}'\mathbf{z} + \delta z^S + c_1(K - \mathbf{1}'\mathbf{z})^2$ . Here,  $c_0 > 0$  is the common processing cost parameter,  $\delta \geq 0$  represents the additional processing cost of S-input due to nonuniformity, and

<sup>2</sup> This is in line with practice in the beef industry where C-input is priced through formula pricing that ties the base price to the spot price, with a specified surcharge for the higher premium content of C-input (e.g., MacDonald 2003), and in line with the pork-hog industry where window contracts are common (e.g., Roe et al. 2004). As in the metals industry (e.g., Kleindorfer and Wu 2003, Geman 2005), the reason for using the spot price as a benchmark for contract prices is so that neither party to the trade then ends up with windfall gains or losses relative to the observable benchmark of the spot market, with consequent incentives for regret and renegeing on the contract.

$c_1 \geq 0$  is a utilization cost parameter. As the total processed input ( $\mathbf{1}'\mathbf{z}$ ) increases, the average variable cost  $C(\mathbf{z})/\mathbf{1}'\mathbf{z}$  decreases.<sup>3</sup>

### 3.4. Production

For each unit of input, there are two possible outputs, and the maximum proportions of these depend on the sources of input. We denote  $a_j^i$  as the fixed proportion of the processed type  $j = \{C, S\}$  input for product  $i = \{1, 2\}$ . We assume  $\mathbf{a}'_1 = (a_1^C, a_1^S) \leq \mathbf{a}'_2 = (a_2^C, a_2^S)$ , i.e., the maximum premium product available from a unit of input is lower than the potential standard product available, whatever the source of the input. We also assume  $a_j^C + a_j^S = s \leq 1$  for  $j \in \{C, S\}$ , i.e., the total yield is identical for both input types, with yield losses from processing ( $s < 1$ ). To capture quality differences in the two input sources, we assume  $a_1^C = a_1^S + \Delta$  and  $a_2^C = a_2^S - \Delta$  for  $\Delta \geq 0$ , where  $\Delta$  denotes the quality premium for C-input, i.e., C-input provides a higher proportion of premium product than S-input.

The firm-specific demand for final outputs is stochastic, price dependent, and represented by the linear inverse-demand functions  $p_1(\mathbf{x}, \tilde{\xi}_1) = \tilde{\xi}_1 + \alpha(\tilde{P}^S - \mu_S) - b_1x_{11} - e(x_{22} + x_{12})$  and  $p_2(\mathbf{x}, \tilde{\xi}_2) = \tilde{\xi}_2 + \alpha(\tilde{P}^S - \mu_S) - b_2(x_{22} + x_{12}) - ex_{11}$ . Here,  $\mathbf{x}' = (x_{11}, x_{22}, x_{12})$  is the production vector,  $e$  represents the cross-price elasticity parameter, and  $b_i$  and  $p_i$  denote own-price slope of the inverse demand function and price for product  $i$ , respectively. The choke price for product  $i$  is  $\tilde{\xi}_i + \alpha(\tilde{P}^S - \mu_S)$ , where  $\alpha$  determines the correlation between the spot price and output prices. A positive (negative)  $\alpha$  implies a positive (negative) correlation. The parameter  $\tilde{\xi}' = (\tilde{\xi}_1, \tilde{\xi}_2)$  is a bivariate random variable with continuous distribution that has bounded expectation  $(\mu_1, \mu_2)$  with covariance matrix  $\Sigma$ , where  $\Sigma_{ii} = \sigma_{\tilde{\xi}_i}^2$  and  $\Sigma_{ij} = \rho_{\xi} \sigma_{\tilde{\xi}_i} \sigma_{\tilde{\xi}_j}$  for  $i \neq j$ , and  $\rho_{\xi}$  denotes the correlation coefficient. We assume that the distributions of  $\tilde{P}^S$  and  $\tilde{\xi}$  are statistically independent.<sup>4</sup> For analytical convenience, we assume  $\alpha < (1 - \omega)/s$ .<sup>5</sup>

<sup>3</sup> Fixed costs are also important elements of the cost structure of processors. They represent payments to capital providers and indirect facility costs. We neglect these in the model development because they do not affect the optimal solution. Fixed costs are reflected in the calibration underlying our numerical results in §6. Decreasing short-term average costs throughout the entire range of feasible input levels are well documented and important for packers in the beef industry (Koontz and Lawrence 2010).

<sup>4</sup> If we let  $\tilde{Y}_i = \tilde{\xi}_i + \alpha(\tilde{P}^S - \mu_S)$  denote the choke price for product  $i$ , then our assumptions here imply a multivariate distribution on  $(Y_1, Y_2, P^S)$  with  $\mu_{Y_i} = \mu_i$ ,  $\sigma_{Y_i} = \sqrt{\sigma_{\tilde{\xi}_i}^2 + \alpha^2 \sigma_S^2}$ ,  $\text{Corr}(Y_i, P^S) = \Psi_i = \alpha(\sigma_S/\sigma_{Y_i})$ , and  $\text{Corr}(Y_1, Y_2) = \Psi_1 \Psi_2 + \rho_{\xi} \sqrt{1 - \Psi_1^2} \sqrt{1 - \Psi_2^2}$ , where  $\text{Corr}$  denotes the correlation coefficient.

<sup>5</sup> This limits the amount of positive correlation between the choke price and spot price we can capture in the model below, but for many applications this is not a tight constraint. For example, for the GIPSA data examined in §6 below, where  $\phi = 0.04$  and  $s = 0.6$ ,

In the production vector  $\mathbf{x}$ ,  $x_{kl}$  denotes the quantity of product  $l$  produced from the capacity ( $\mathbf{a}'_k \mathbf{z} = a_k^C z^C + a_k^S z^S$ ) dedicated to product  $k$ . Because the first product is premium product, we assume that  $b_1 \geq b_2 \geq 0$ , i.e., the first product demand is less responsive to changes in price than the second product. In particular, we assume  $b_2 \leq b_1(a_1^S/a_2^S)$ .<sup>6</sup>

We allow for two different substitution channels for production. There exists *downward product substitution*: the firm can produce standard product using the premium product yield, and not vice versa. We assume that the firm uses a market-clearing pricing strategy; i.e., available input is processed into one or other of the two final products, and price is adjusted in profit-maximizing fashion to sell all finished products. There is also *demand substitution* through the cross-price elasticity parameter  $e$ . We assume that outputs are substitutes so that the price of each product is decreasing in the price of the other product ( $e \geq 0$ ), and this cross-price effect is lower than the own-price effect ( $e \leq \min(b_1, b_2)$ ). When  $b_1 = b_2 = e = 0$ , we have the special case where the firm is a pure price taker.

### 3.5. The Firm's Decision Problem

We model the firm's decision problem as a two-stage stochastic recourse problem. In Stage 0, the firm decides on the volume of C-input ( $Q^C$ ) to contract, facing spot price and product market uncertainties. At Stage 1,  $P^S$  and  $\xi$  are realized, and  $Q^C$  is delivered to the firm. The firm then decides on its spot market purchases ( $Q^S$ ), as well as on the volume of input to process from C- and S-input ( $z^C \leq Q^C$  and  $z^S \leq Q^S$ , respectively). This decision also implies the firm's spot market sales, namely, ( $Q^C + Q^S - z^C - z^S$ ). Finally, the firm decides on the production quantities of the two products that either come from their own product yield ( $x_{11}, x_{22}$ ), or through substitution of the premium product yield to produce standard product ( $x_{12}$ ). The objective of the firm is to maximize its expected total profit at Stage 0.

We now formulate the firm's decision problem starting from Stage 1:

$$\max_{Q^S, z, x} \left\{ -Q^C [\max(\min(u, P^S + \nu), l)] - Q^S (P^S + t) \right. \\ \left. + (1 - \omega) P^S [Q^C + Q^S - \mathbf{1}'\mathbf{z}] \right.$$

this constraint implies an upper bound of 1.6 on  $\alpha$ . For estimated values of the variance of spot and product prices, this implies a maximum correlation of 0.75, well above that required to model realistic correlations in the beef industry.

<sup>6</sup> This is an appropriate assumption for beef markets, where price sensitivity is considerably higher for premium products than for standard products. However, all of the results related to the characterization of the optimal solution hold for (i)  $\mathbf{a}_1 > \mathbf{a}_2$ , (ii)  $\mathbf{a}_1 \leq \mathbf{a}_2$  and  $b_2 \leq b_1(a_1^S/a_2^S)$ , and (iii)  $\mathbf{a}_1 \leq \mathbf{a}_2$  and  $b_2 \in [b_1(a_1^S/a_2^S), b_1)$  with an additional restriction on the demand substitution parameter,  $e \in [(b_2 a_2^S - b_1 a_1^S)/(a_2^S - a_1^S), b_2)$ .

$$\begin{aligned} & - [c_0 \mathbf{1}'\mathbf{z} + \delta z^S + c_1 (K - \mathbf{1}'\mathbf{z})^2] \\ & + x_{11} (\xi_1 + \alpha (P^S - \mu_S) - b_1 x_{11}) \\ & + (x_{22} + x_{12}) (\xi_2 + \alpha (P^S - \mu_S) - b_2 (x_{22} + x_{12})) \\ & - 2e (x_{22} + x_{12}) x_{11} \end{aligned} \quad (1)$$

$$\text{s.t. } z^C \leq Q^C, \quad z^S \leq Q^S, \quad \mathbf{1}'\mathbf{z} \leq K,$$

$$x_{11} + x_{12} = \mathbf{a}'_1 \mathbf{z}, \quad x_{22} = \mathbf{a}'_2 \mathbf{z},$$

$$Q^S \geq 0, \quad \mathbf{z} \geq \mathbf{0}, \quad \mathbf{x} \geq \mathbf{0}.$$

In (1), the first two terms represent the total procurement cost of the firm. The third term is the revenue from spot market sales, and the fourth term is the firm's total processing cost. The final terms in the objective function denote the sales revenue from the product markets. The first two constraints ensure that the firm does not process more than the available input of each type. The third constraint is the firm's plant size constraint. The fourth and the fifth constraints represent the available yield for each output under market-clearing pricing.<sup>7</sup> Let  $\Pi(Q^C; P^S, \xi)$  denote the optimal Stage 1 profit for a given  $Q^C$ .

Anticipating these decisions, at Stage 0, the firm solves for the optimal C-input to contract,  $Q^{C*}$ , to maximize its expected profit:  $V^* = \max_{Q^C \geq 0} V(Q^C) = \mathbb{E}[\Pi(Q^C; P^S, \xi)]$ , where the expectation is taken over  $\tilde{P}^S$  and  $\tilde{\xi}$ .

## 4. The Optimal Strategy

In this section, we describe the optimal solution for the firm's procurement, processing, and production decisions. We solve the problem by backward induction starting from Stage 1. All the proofs are relegated to the online technical appendix (available at [http://ink.library.smu.edu.sg/lkcsb\\_research/1801/](http://ink.library.smu.edu.sg/lkcsb_research/1801/)).

### 4.1. Stage 1: Spot Market

To find the optimal solution for (1), we first solve for the firm's optimal output of products 1 and 2,  $\mathbf{x}' = (x_{11}, x_{22}, x_{12})$ , given the vector of processed inputs  $\mathbf{z}' = (z^C, z^S)$ :

$$\begin{aligned} \max_{\mathbf{x}} \quad & \{ x_{11} (\xi_1 + \alpha (P^S - \mu_S) - b_1 x_{11}) + (x_{22} + x_{12}) \\ & \cdot (\xi_2 + \alpha (P^S - \mu_S) - b_2 (x_{22} + x_{12})) - 2e (x_{22} + x_{12}) x_{11} \} \\ \text{s.t. } \quad & x_{11} + x_{12} = \mathbf{a}'_1 \mathbf{z}, \quad x_{22} = \mathbf{a}'_2 \mathbf{z}, \quad \mathbf{x} \geq \mathbf{0}. \end{aligned} \quad (2)$$

<sup>7</sup> It is theoretically possible that a profit-maximizing firm could engage in pure waste to attempt to affect the price of its products so that a more general model would allow these processing capacity constraints to hold as inequalities. However, this theoretical possibility is not of interest when final product markets are highly competitive, as they are in beef for example, where the firm-specific price elasticity of demand is very high. Thus, to avoid uninteresting complications, we treat these constraints as equalities.

The key decision here is the optimal product substitution level, i.e., the allocation between the two final products of the available premium product yield  $\mathbf{a}'_1\mathbf{z}$ .

**PROPOSITION 1.** *The unique optimal production vector  $\mathbf{x}^*(\mathbf{z}, \xi, \mathbf{P}^S)' = (x_{11}^*, x_{22}^*, x_{12}^*)$  for a given processed input vector  $\mathbf{z}' = (z^C, z^S)$  is given by*

$$\mathbf{x}^*(\mathbf{z}, \xi, \mathbf{P}^S)' = \begin{cases} (\mathbf{a}'_1\mathbf{z}, \mathbf{a}'_2\mathbf{z}, 0) & \text{if } \xi \in \Gamma^1, \\ \left( \frac{(b_2 - e)}{b_1 + b_2 - 2e} s\mathbf{1}'\mathbf{z} + \frac{\xi_1 - \xi_2}{2(b_1 + b_2 - 2e)}, \mathbf{a}'_2\mathbf{z}, \right. \\ \left. \frac{(b_1 - e)}{b_1 + b_2 - 2e} \mathbf{a}'_1\mathbf{z} - \frac{(b_2 - e)}{b_1 + b_2 - 2e} \mathbf{a}'_2\mathbf{z} - \frac{\xi_1 - \xi_2}{2(b_1 + b_2 - 2e)} \right) & \text{if } \xi \in \Gamma^2, \\ (0, \mathbf{a}'_2\mathbf{z}, \mathbf{a}'_1\mathbf{z}) & \text{if } \xi \in \Gamma^3, \end{cases}$$

where

$$\begin{aligned} \Gamma^1 &\doteq \{\xi: \xi \geq 0, \xi_2 \leq \xi_1 - 2[(b_1 - e)\mathbf{a}'_1\mathbf{z} - (b_2 - e)\mathbf{a}'_2\mathbf{z}]\}, \\ \Gamma^2 &\doteq \{\xi: \xi \geq 0, \xi_2 > \xi_1 - 2[(b_1 - e)\mathbf{a}'_1\mathbf{z} - (b_2 - e)\mathbf{a}'_2\mathbf{z}], \\ &\quad \xi_2 < \xi_1 + 2(b_2 - e)s\mathbf{1}'\mathbf{z}\}, \\ \Gamma^3 &\doteq \{\xi: \xi \geq 0, \xi_2 \geq \xi_1 + 2(b_2 - e)s\mathbf{1}'\mathbf{z}\}. \end{aligned}$$

The optimal sales revenue in the product markets,  $\pi^*(\mathbf{z}, \xi, \mathbf{P}^S)$ , is characterized by

$$\begin{aligned} \pi^1(\mathbf{a}'_1\mathbf{z}, \mathbf{a}'_2\mathbf{z}, \xi, P^S) &\doteq \xi_1 \mathbf{a}'_1\mathbf{z} - b_1(\mathbf{a}'_1\mathbf{z})^2 + \xi_2 \mathbf{a}'_2\mathbf{z} - b_2(\mathbf{a}'_2\mathbf{z})^2 \\ &\quad - 2e(\mathbf{a}'_1\mathbf{z})(\mathbf{a}'_2\mathbf{z}) + \alpha(P^S - \mu_S)s\mathbf{1}'\mathbf{z} \quad \text{if } \xi \in \Gamma^1, \\ \pi^2(\mathbf{a}'_1\mathbf{z}, \mathbf{a}'_2\mathbf{z}, \xi, P^S) &\doteq \frac{(\xi_1 - \xi_2)^2}{4(b_1 + b_2 - 2e)} + \frac{\xi_1(b_2 - e) + \xi_2(b_1 - e)}{b_1 + b_2 - 2e} s\mathbf{1}'\mathbf{z} \\ &\quad - \frac{(b_1 b_2 - e^2)}{b_1 + b_2 - 2e} (s\mathbf{1}'\mathbf{z})^2 + \alpha(P^S - \mu_S)s\mathbf{1}'\mathbf{z} \quad \text{if } \xi \in \Gamma^2, \\ \pi^3(\mathbf{a}'_1\mathbf{z}, \mathbf{a}'_2\mathbf{z}, \xi, P^S) &\doteq \xi_2 s\mathbf{1}'\mathbf{z} - b_2(s\mathbf{1}'\mathbf{z})^2 + \alpha(P^S - \mu_S)s\mathbf{1}'\mathbf{z} \quad \text{if } \xi \in \Gamma^3. \end{aligned}$$

When  $\xi_1$  is sufficiently greater than  $\xi_2$ , the first market is highly profitable compared to the second market, and the firm optimally allocates all the available premium product yield  $\mathbf{a}'_1\mathbf{z}$  to the first product ( $\xi \in \Gamma^1$ ). We denote this as the *no product substitution regime*. Similarly, when  $\xi_2$  is sufficiently greater than  $\xi_1$ , the second market is highly profitable compared to the first market, and the firm optimally allocates all the available premium product yield  $\mathbf{a}'_1\mathbf{z}$  to the second product ( $\xi \in \Gamma^3$ ). We denote this as the *full product substitution regime*. If the difference between  $\xi_1$  and  $\xi_2$  is moderate, then the firm optimally allocates some part

of  $\mathbf{a}'_1\mathbf{z}$  to both products ( $\xi \in \Gamma^2$ ). We denote this as the *partial product substitution regime*. The allocation to the standard product decreases as  $(\xi_1 - \xi_2)$  increases.

For notational convenience, we define  $\pi^k(\mathbf{a}'_1\mathbf{z}, \mathbf{a}'_2\mathbf{z}, \xi, P^S)$  as the optimal sales revenue under a type  $k$  product substitution regime (or  $\xi \in \Gamma^k$ ) for  $k \in \{1, 2, 3\}$  for given product  $i$  yields  $\mathbf{a}'_i\mathbf{z}$ ,  $i \in \{1, 2\}$ . Here, 1 represents no substitution, 2 represents partial substitution, and 3 represents full substitution.

The firm cannot generate revenue by selling S-input back to the spot market (because there are transaction costs in both spot procurement ( $t$ ) and spot sales ( $\phi$ )). Thus,  $Q^{S*} = z^{S*}$ : Any input the firm procures from the spot market is processed. Because the optimal production vector is uniquely defined by  $\mathbf{z}$ , we can optimize the Stage 1 problem over the processing vector  $\mathbf{z}' = (z^C, z^S)$ . We now state an important property of the optimal solution that will enable us to simplify further the decision problem in (1).

**PROPOSITION 2.**  $(Q^C - z^{C*}) \times z^{S*} = 0$ .

The firm only processes S-input after all available C-input has been used. This is because C-input is preferred over S-input because it has a lower procurement cost ( $\omega \geq 0, t \geq 0$ ), a lower processing cost ( $\delta \geq 0$ ), and a higher proportion of the premium product ( $\Delta \geq 0$ ) that is valuable under the no product substitution regime as follows from Proposition 1.

Using Proposition 2, we can redefine the Stage 1 decision problem in (1) as a single-variable optimization problem. We relegate the detailed characterization of this equivalent formulation to §A of the online technical appendix. We only provide the highlights of this formulation. Let  $z$  denote the total processing amount, and let  $\Lambda(z)$  denote the Stage 1 objective function. In the optimal solution, we have  $z^{C*} = \min(z^*, Q^C)$  and  $z^{S*} = (z^* - Q^C)^+$ .

To obtain the equivalent formulation, we first define

$$\begin{aligned} \Lambda^{k,C}(z) &= -Q^C[\max(\min(u, P^S + v), u)] \\ &\quad + (1 - \omega)P^S[Q^C - z] - c_0 z - c_1(K - z)^2 \\ &\quad + \pi^k(a_1^C z, a_2^C z, \xi, P^S), \\ \Lambda^{k,S}(z) &= -Q^C[\max(\min(u, P^S + v), u)] \\ &\quad - (z - Q^C)(P^S + t) - c_0 z - \delta(z - Q^C) - c_1(K - z)^2 \\ &\quad + \pi^k((a_1^C - a_1^S)Q^C + a_1^S z, (a_2^C - a_2^S)Q^C + a_2^S z, \xi, P^S), \end{aligned}$$

for  $k \in \{1, 2, 3\}$ , where  $\Lambda^{k,C}$  represents the objective function when the firm only uses C-input for processing, and the optimal production belongs to a type  $k$  product substitution regime. Notice that the argument of  $\pi^k$  is given by  $a_i^C z$  because we are only processing C-input. Similarly,  $\Lambda^{k,S}$  denotes the objective function when the firm processes S-input, and the optimal

product substitution regime is of type  $k$ . The argument of  $\pi^k$  is given by  $(a_i^C - a_i^S)Q^C + a_i^S z$  because the first  $Q^C$  units of  $z$  are C-inputs. The Stage 1 objective function  $\Lambda(z)$  is a combination of  $\Lambda^{k,j}$  for  $k \in \{1, 2, 3\}$  and  $j \in \{C, S\}$ , and  $\Lambda(z)$  is strictly concave in  $z$  (see the online technical appendix).

There exists a six-region partitioning of  $(\xi_1, \xi_2)$  space such that the formulation of the Stage 1 problem takes a unique form in each of these regions. These regions correspond to each of the three product substitution regimes and which of the processed inputs, C-input or S-input, is used under these substitution regimes. The six regions are defined as follows.

$\Omega^1$ : no substitution for C- and S-input;

$\Omega^2$ : no substitution for C-input, no and partial substitution for S-input;

$\Omega^3$ : no and partial substitution for C-input, partial substitution for S-input;

$\Omega^4$ : full and partial substitution for C-input, partial substitution for S-input;

$\Omega^5$ : full substitution for C-input, full and partial substitution for S-input;

$\Omega^6$ : full substitution for C- and S-input where

$$\Omega^1 \doteq \{\xi: \xi \geq 0, \xi_2 \leq \xi_1 - 2[(b_1 - e)a_1^C - (b_2 - e)a_2^C]K, \\ + 2[(b_1 - e)(a_1^C - a_1^S) - (b_2 - e)(a_2^C - a_2^S)](K - Q^C)^+\},$$

$$\Omega^2 \doteq \{\xi: \xi \geq 0, \xi_2 \leq \xi_1 - 2[(b_1 - e)a_1^C \\ - (b_2 - e)a_2^C] \min(Q^C, K), \xi_2 > \xi_1 - 2[(b_1 - e)a_1^C \\ - (b_2 - e)a_2^C]K + 2[(b_1 - e)(a_1^C - a_1^S) \\ - (b_2 - e)(a_2^C - a_2^S)](K - Q^C)^+\},$$

$$\Omega^3 \doteq \{\xi: \xi \geq 0, \xi_2 > \xi_1 - 2[(b_1 - e)a_1^C \\ - (b_2 - e)a_2^C] \min(Q^C, K), \xi_2 \leq \xi_1\},$$

$$\Omega^4 \doteq \{\xi: \xi \geq 0, \xi_2 > \xi_1, \xi_2 \leq \xi_1 + 2(b_2 - e)S \min(Q^C, K)\},$$

$$\Omega^5 \doteq \{\xi: \xi \geq 0, \xi_2 > \xi_1 + 2(b_2 - e)S \min(Q^C, K), \\ \xi_2 \leq \xi_1 + 2(b_2 - e)SK\},$$

$$\Omega^6 \doteq \{\xi: \xi \geq 0, \xi_2 > \xi_1 + 2(b_2 - e)SK\}.$$

The above structure is intuitive: As the difference between premium and standard product market profitability decreases, i.e.,  $\xi_1 - \xi_2$  decreases, the firm moves from no substitution for either input type (in  $\Omega^1$ ) to various degrees of partial substitution (in  $\Omega^{2,3,4,5}$ ) to full substitution for either input type (in  $\Omega^6$ ) in the product markets.

To provide further intuition, consider the special case in which the firm is a pure price taker ( $b_1 = b_2 = e = 0$ ). In this case, the above six regions collapse into two ( $\Omega^1$  and  $\Omega^6$ ): the firm uses no product substitution for either input type when  $\xi \in \Omega^1 = \{\xi: \xi \geq 0, \xi_2 \leq \xi_1\}$  and full product substitution when  $\xi \in \Omega^6 = \{\xi: \xi \geq 0, \xi_2 > \xi_1\}$ .

The optimal processing decision  $z^*$  for each of the  $\Omega^{(\cdot)}$  regions is technical, but straightforward given the quadratic objective function and linear constraints. In each of these regions, the optimal processing decision  $z^*$  is unique and is characterized by a number of spot price thresholds. In particular, eight spot price thresholds (denoted by  $\bar{P}^{(\cdot)}$ ) characterize  $z^*$  for  $\xi \in \Omega^{123}$ , and another eight spot price thresholds (denoted by  $\underline{P}^{(\cdot)}$ ) characterize  $z^*$  for  $\xi \in \Omega^{456}$ . As shown in §B of the online technical appendix, the two sets of eight spot price thresholds each have a fixed order, but they appear in different combinations for the optimal solution for each of the  $\Omega^{(\cdot)}$  regions.

As an example, consider  $\xi \in \Omega^1$ , where the firm uses no substitution regime for either input. In this region,  $z^*$  is characterized by

$$z^* = \begin{cases} 0 & \text{if } P^S \geq \bar{P}^0, \\ z_{1,C}^* = \frac{(1 - \omega - \alpha s)(\bar{P}^0 - P^S)}{2[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2ea_1^C a_2^C + c_1]} & \text{if } \bar{P}^0 > P^S \geq \bar{P}^1(\min(Q^C, K)), \\ \min(Q^C, K) & \text{if } \bar{P}^1(\min(Q^C, K)) > P^S \geq \bar{P}^4(\min(Q^C, K)), \\ z_{1,S}^* = \min(Q^C, K) + \frac{(\bar{P}^4 - P^S)(1 - \alpha s)}{2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2ea_1^S a_2^S + c_1]} & \text{if } \bar{P}^4(\min(Q^C, K)) > P^S \geq \bar{P}^5(K), \\ K & \text{if } \bar{P}^5(K) > P^S, \end{cases} \tag{3}$$

where

$$\bar{P}^0 \doteq \frac{\xi_1 a_1^C + \xi_2 a_2^C + 2c_1 K - c_0 - \alpha s \mu_S}{1 - \omega - \alpha s},$$

$$\bar{P}^1(\min(Q^C, K))$$

$$\doteq (1 - \omega - \alpha s)^{-1} [\xi_1 a_1^C + \xi_2 a_2^C + 2c_1 K - c_0 - \alpha s \mu_S \\ - 2[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2ea_1^C a_2^C + c_1] \min(Q^C, K)],$$

$$\bar{P}^4(\min(Q^C, K))$$

$$\doteq (1 - \alpha s)^{-1} [\xi_1 a_1^S + \xi_2 a_2^S + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S \\ - 2Q^C \Delta[(b_1 - e)a_1^S + (b_2 - e)a_2^S] \\ - 2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2ea_1^S a_2^S + c_1] \min(Q^M, K)],$$

$$\bar{P}^5(K) \doteq (1 - \alpha s)^{-1} [\xi_1 a_1^S + \xi_2 a_2^S + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S \\ - 2Q^C \Delta[(b_1 - e)a_1^S + (b_2 - e)a_2^S] \\ - 2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2ea_1^S a_2^S + c_1]K].$$

In  $\bar{P}^{(\cdot)}(y)$ , the argument  $y$  refers to the last term in the definition of the threshold. Here,  $z_{i,j}^*$  is the unique

solution to  $(\partial/\partial z)\Lambda^{k,j} = 0$  for  $k \in \{1, 2, 3\}$  and  $j \in \{C, S\}$ . The intuition behind (3) is straightforward: As  $P^S$  decreases, the firm processes more units (starting from C-input).<sup>8</sup> This is because spot sales become less profitable, and spot procurement becomes cheaper. The exact form of the optimal solution is determined by comparing the marginal revenue of processing an additional unit of C- or S-input with the corresponding spot option cost (sale or procurement), leading to the various price breakpoints indicated.

#### 4.2. Stage 0: Contract Market

At this stage the firm chooses the volume of C-input to contract to maximize its expected profit in the presence of spot price and product market uncertainties. The following proposition characterizes the optimal contracting decision  $Q^{C*}$  with the assumption that  $\bar{P}^S$  follows a normal distribution with  $(\mu_S, \sigma_S)$ . The normality assumption is useful in delineating the intuition behind the technical statements. The characterization for a general  $\bar{P}^S$  distribution is structurally the same and is provided in §C of the online technical appendix.

**PROPOSITION 3.** *The optimal volume of C-input is never higher than plant size ( $Q^{C*} \leq K$ ) and is characterized by the following first-order condition:*

$$\begin{aligned} & \frac{\partial}{\partial Q^C} V \\ &= - \left[ u + \sigma_S \left( L \left( \frac{l - \nu - \mu_S}{\sigma_S} \right) - L \left( \frac{u - \nu - \mu_S}{\sigma_S} \right) \right) \right] \\ & \quad + \mu_S(1 - \omega) \\ & \quad + \sigma_S \mathbb{E} \left[ (1 - \omega - \alpha s) L \left( \frac{\bar{P}^1(Q^C) - \mu_S}{\sigma_S} \right) \right. \\ & \quad \quad \left. - (1 - \alpha s) L \left( \frac{\bar{P}^4(Q^C) - \mu_S}{\sigma_S} \right) \Big| \tilde{\xi} \in \Omega^{12} \right] \Pr(\tilde{\xi} \in \Omega^{12}) \\ & \quad + \sigma_S \mathbb{E} \left[ (1 - \omega - \alpha s) L \left( \frac{P^3(Q^C) - \mu_S}{\sigma_S} \right) \right. \\ & \quad \quad \left. - (1 - \alpha s) L \left( \frac{P^6(Q^C) - \mu_S}{\sigma_S} \right) \Big| \tilde{\xi} \in \Omega^{34} \right] \Pr(\tilde{\xi} \in \Omega^{34}) \\ & \quad + \sigma_S \mathbb{E} \left[ (1 - \omega - \alpha s) L \left( \frac{P^1(Q^C) - \mu_S}{\sigma_S} \right) \right. \\ & \quad \quad \left. - (1 - \alpha s) L \left( \frac{P^4(Q^C) - \mu_S}{\sigma_S} \right) \Big| \tilde{\xi} \in \Omega^{56} \right] \Pr(\tilde{\xi} \in \Omega^{56}) \end{aligned}$$

<sup>8</sup> This result depends on our assumption  $\alpha < (1 - \omega)/s$ , which insures that the cost effect of  $P^S$  (because of spot procurement for S-input and opportunity loss of spot resale for C-input) dominates the revenue effect (because of the signaling effect of  $P^S$  for product market prices through the correlation parameter  $\alpha$ ) for any input type, so that a higher spot price has a negative impact on the value of processing. The results for  $\alpha > (1 - \omega)/s$  are available from the authors.

$$\begin{aligned} & - \Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S] \\ & \cdot \mathbb{E}[(\sigma_S(1 - \alpha s)(L((\bar{P}^4(Q^C) - \mu_S)/\sigma_S)) \\ & \quad - L((\bar{P}^5(\min(I(S), K)) - \mu_S)/\sigma_S))] \cdot (b_1(a_1^S)^2 + b_2(a_2^S)^2 \\ & \quad + 2ea_1^S a_2^S + c_1)^{-1} \Big| \tilde{\xi} \in \Omega^{12}] \Pr(\tilde{\xi} \in \Omega^{12}), \end{aligned} \quad (4)$$

where  $L(\eta) = \int_{-\infty}^{\eta} (\eta - z)\phi(z) dz$  is the standard-normal loss function,

$$I(S) \doteq \frac{(\xi_1 - \xi_2)/2 - Q^C \Delta(b_1 + b_2 - 2e)}{(b_1 - e)a_1^S - (b_2 - e)a_2^S},$$

$$P^3(Q^C) = \bar{P}^3(Q^C) = \underline{P}^3(Q^C)$$

$$\begin{aligned} & \doteq ([\xi_1(b_2 - e) + \xi_2(b_1 - e)]s / (b_1 + b_2 - 2e) + 2c_1K - c_0 \\ & \quad - \alpha s \mu_S - 2[(b_1 b_2 - e^2)s^2 / (b_1 + b_2 - 2e) + c_1]Q^C) \\ & \quad \cdot (1 - \omega - \alpha s)^{-1}, \end{aligned}$$

$$P^6(Q^C) = \bar{P}^6(Q^C) = \underline{P}^6(Q^C)$$

$$\begin{aligned} & \doteq ([\xi_1(b_2 - e) + \xi_2(b_1 - e)]s / (b_1 + b_2 - 2e) + 2c_1K - c_0 \\ & \quad - t - \delta - \alpha s \mu_S - 2[(b_1 b_2 - e^2)s^2 / (b_1 + b_2 - 2e) + c_1]Q^C) \\ & \quad \cdot (1 - \alpha s)^{-1}, \end{aligned}$$

$$\underline{P}^1(Q^C)$$

$$\doteq \frac{\xi_2 s + 2c_1 K - c_0 - \alpha s \mu_S - 2[b_2 s^2 + c_1] \min(I, Q^C, K)}{1 - \omega - \alpha s},$$

$$\underline{P}^4(Q^C)$$

$$\doteq \frac{\xi_2 s + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S - 2[b_2 s^2 + c_1]Q^C}{1 - \alpha s},$$

and thresholds  $\bar{P}^1, \bar{P}^4$ , and  $\bar{P}^5$  are as given in (3). We have  $Q^{C*} = 0$  if  $(\partial V / \partial Q^C)|_{0+} \leq 0$  and  $Q^{C*} = K$  if  $(\partial V / \partial Q^C)|_{K-} \geq 0$ ; otherwise, it is the solution to  $\partial V / \partial Q^C = 0$ .

The first term in (4) is the expected marginal contract procurement cost. We note here that with  $l \rightarrow -\infty$  and  $u \rightarrow \infty$ , this term equals  $\mu_S + \nu$ . This is the expected unit cost of C-input, including the adjustment  $\nu$  to account for differences between C-input and S-input. When  $l = u$ , this term equals  $u$ , and the resulting contract price is independent of  $P^S$ , which represents the case of a fixed forward contract.

To understand the remaining terms in (4), let us first consider the special case in which the firm can only sell excess C-input into the spot market, but cannot procure S-input.

**COROLLARY 1.** *If the firm does not have access to spot procurement, i.e.,  $t \rightarrow \infty$ , the optimality condition in (4) is given by*

$$\begin{aligned} & \frac{\partial V}{\partial Q^C} = - \left[ u + \sigma_S \left( L \left( \frac{l - \nu - \mu_S}{\sigma_S} \right) - L \left( \frac{u - \nu - \mu_S}{\sigma_S} \right) \right) \right] \\ & \quad + \mu_S(1 - \omega) + \sigma_S(1 - \omega - \alpha s) \\ & \quad \cdot \sum_{l=1}^6 \mathbb{E} \left[ L \left( \frac{\hat{P}^l - \mu_S}{\sigma_S} \right) \Big| \tilde{\xi} \in \Omega^l \right] \Pr(\tilde{\xi} \in \Omega^l), \end{aligned} \quad (5)$$

where  $\hat{P}^1 = \hat{P}^2 = \bar{P}^1(Q^C)$ ,  $\hat{P}^3 = \hat{P}^4 = P^3(Q^C)$ , and  $\hat{P}^5 = \hat{P}^6 = \underline{P}^1(Q^C)$ .

The sum of  $\mu_s(1 - \omega)$  and the final term in (5) is the expected marginal revenue of an additional unit of C-input (without spot procurement, but allowing spot sale of the C-input). At Stage 1, the firm has two options for C-input, spot sale or processing. Therefore, the indicated expected marginal revenue is the maximum of these two options. This is represented as the sum of the expected marginal profit from a spot sale ( $\mu_s(1 - \omega)$ ) and the marginal profit over a spot sale from processing the C-input. In the absence of spot procurement, the additional unit of C-input is processed only if the firm optimally processes all the available C-input at Stage 1, i.e.,  $z^* = Q^C$ . From (3), it can be shown that the marginal profit of processing at  $z = Q^C$  is  $(1 - \omega - \alpha s)(P^{(\cdot)} - \bar{P}^5)^+$ . The form of  $P^{(\cdot)}$  depends on the product substitution regime used with C-input processing and is different across the  $\Omega^{(\cdot)}$  regions. This explains the  $\sigma_s \mathbb{E}[(1 - \omega - \alpha s)L(\cdot)]$  terms in (4).

Access to spot procurement has a negative impact on the marginal revenue of an additional C-input. In (4), this impact is captured by the expression in the last line and the second terms ( $-\sigma_s(1 - \alpha s)\mathbb{E}[L(\cdot)]$ ) in the second, third, and fourth lines. Because C-input is preferred over S-input for processing, the additional unit of C-input is always processed for  $z^* \geq Q^C$ . For spot price realizations inducing  $z^* > Q^C$ , the firm replaces the first unit of S-input with the additional unit of C-input. Therefore, the firm loses the marginal profit of processing the first unit of S-input. From (3), it can be shown that the marginal profit of processing the first unit of S-input is  $(1 - \alpha s)(P^{(\cdot)} - \bar{P}^5)^+$ . The form of  $P^{(\cdot)}$  depends on the product substitution regime used with S-input processing and is different within  $\Omega^{(\cdot)}$  regions. This explains  $-\sigma_s \mathbb{E}[(1 - \alpha s)L(\cdot)]$  terms in (4).

The expression in the last line of (4) is the impact of an additional unit of C-input on all S-inputs. When the firm operates under the no product substitution regime for S-input, i.e., for  $\xi \in \Omega^{12}$ , the marginal profit of processing an S-input is given by

$$\begin{aligned} \frac{\partial}{\partial z} \Lambda^{1,S} &= -c_0 + 2c_1(K - z) - P^S - t - \delta \\ &+ a_1^S(\xi_1 + \alpha(P^S - \mu_s)) + a_2^S(\xi_2 + \alpha(P^S - \mu_s)) \\ &- 2\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S]Q^C \\ &- 2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2ea_1^S a_2^S]z. \end{aligned} \quad (6)$$

It follows from (6) that an increase in  $Q^C$  decreases the marginal profit of processing S-input. The additional C-input is processed before any S-input and alters the output prices by providing a higher (lower) yield of premium (standard) product than S-input as  $\Delta \geq 0$ .

This decreases (increases) the marginal production revenue of the premium (standard) product. Because  $[(b_1 - e)a_1^S - (b_2 - e)a_2^S] \geq 0$ , the net impact is that the marginal revenue of processing S-input decreases. This effect does not exist if there is no quality difference, i.e.,  $\Delta = 0$ , or if the firm is a price taker, i.e.,  $b_1 = b_2 = e = 0$ . This effect also does not exist under the other product substitution regimes because the firm is indifferent between C- and S-input with respect to production revenues.<sup>9</sup> When this effect exists, it is relevant for all S-input processed under the no product substitution regime. In fact, if we define  $z_1^*$  as the optimal processing quantity under the no product substitution regime, the last expression in (4) is equivalent to  $\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S]\mathbb{E}[(z_1^* - Q^C)^+]$ . Here  $\mathbb{E}[(z_1^* - Q^C)^+]$  denotes the optimal expected volume of S-input processed under the no substitution regime.

We close this section with the optimality condition in (4) for the interesting special case where the firm is a pure price taker in its product markets.

**COROLLARY 2.** *If the firm is a price taker in the product markets, i.e.,  $b_1 = b_2 = e = 0$ , the optimality condition in (4) is given by*

$$\begin{aligned} &\frac{\partial V}{\partial Q^C} \\ &= -\left[ u + \sigma_s \left( L\left(\frac{l - v - \mu_s}{\sigma_s}\right) - L\left(\frac{u - v - \mu_s}{\sigma_s}\right) \right) \right] + \mu_s(1 - \omega) \\ &+ \sigma_s \mathbb{E} \left[ (1 - \omega - \alpha s) L\left(\frac{\bar{P}^1(Q^C) - \mu_s}{\sigma_s}\right) \right. \\ &\quad \left. - (1 - \alpha s) L\left(\frac{\bar{P}^4(Q^C) - \mu_s}{\sigma_s}\right) \Big| \tilde{\xi} \in \Omega^1 \right] \Pr(\tilde{\xi} \in \Omega^1) \\ &+ \sigma_s \mathbb{E} \left[ (1 - \omega - \alpha s) L\left(\frac{P^1(Q^C) - \mu_s}{\sigma_s}\right) \right. \\ &\quad \left. - (1 - \alpha s) L\left(\frac{P^4(Q^C) - \mu_s}{\sigma_s}\right) \Big| \tilde{\xi} \in \Omega^6 \right] \Pr(\tilde{\xi} \in \Omega^6), \end{aligned} \quad (7)$$

where  $\Omega^1 = \{\xi: \xi \geq 0, \xi_2 \leq \xi_1\}$  is the region of no substitution for either input, and  $\Omega^6 = \{\xi: \xi \geq 0, \xi_2 > \xi_1\}$  is the region of full substitution for both C- and S-input.

## 5. Analysis of Window Contracts

This section describes comparative statics results for the above model focusing on the impact of the spot price variability  $\sigma_s$  (§5.1) and the correlation parameter  $\alpha$  (§5.2) on the optimal expected profit and the

<sup>9</sup> As follows from Proposition 1, the optimal sales revenue from product markets depends on fixed proportions  $a_1$  and  $a_2$  only under the no product substitution regime, and depends only on the total usable input  $s1'z$  under the other substitution regimes.

optimal procurement portfolio (the optimal contract volume and the expected spot procurement at the optimal solution) of the firm. Our managerial insights are summarized in §5.3. We continue to assume  $P^S$  to follow a normal distribution.

### 5.1. Impact of Spot Price Variability $\sigma_S$

We first analyze the impact of the spot price variability  $\sigma_S$  on the optimal expected profit.

**PROPOSITION 4.** *The optimal expected profit of the firm,  $V^*$ , increases in  $\sigma_S$  if  $l = u$ , or  $l \rightarrow -\infty, u \rightarrow \infty$ , or  $\mu_S + \nu > (l + u)/2$ .*

An increase in  $\sigma_S$  impacts both the expected revenue and the expected contract procurement cost of the firm. On the revenue side, the firm benefits from spot price variability because it buys cheap when the spot price is low and resells to the spot when the spot price is high. On the cost side, a higher spot price variability increases the expected contract procurement cost only if the window contract caps upside variability in contract prices less than downside variability relative to mean contract procurement prices:  $\mu_S + \nu < (l + u)/2$ .<sup>10</sup> Therefore, if the firm uses a fixed forward contract ( $l = u$ ), or an unconstrained contract ( $l \rightarrow -\infty, u \rightarrow \infty$ ), or any contract with higher upside protection than foregone downside contract procurement savings ( $\mu_S + \nu > (l + u)/2$ ), then the optimal expected firm profit increases in  $\sigma_S$ .

To analyze the impact of  $\sigma_S$  on the optimal procurement portfolio, we will focus on the pure price-taker special case of our model as presented in Corollary 2. In this case, the expected marginal cost of C-input is given by

$$u + \sigma_S \left( L \left( \frac{l - \nu - \mu_S}{\sigma_S} \right) - L \left( \frac{u - \nu - \mu_S}{\sigma_S} \right) \right),$$

and the expected marginal revenue of C-input is given by the value of spot and processing options at Stage 1. In particular, the marginal revenue at Stage 1 is characterized by the processing option when the spot price is in a certain window (the processing window); and outside this window, it is characterized by the opportunity gain from not using spot procurement when the spot price is lower and spot sale revenue when the spot price is higher.

Implicit differentiation of the optimality condition in (7) yields the following expression characterizing

<sup>10</sup> Recall that the unit price of C-input on the day is  $[\max(\min(u, P^S + \nu), u)]$  with  $l \leq u$ . Thus, when  $l - \nu$  is closer to the mean spot price  $\mu_S$  than  $u - \nu$ , the firm cannot benefit from low  $P^S$  realizations as much to compensate for the negative impact of high  $P^S$  realizations.

the sign of the impact of  $\sigma_S$  on the optimal contract volume:

$$\begin{aligned} & - \left( \phi \left( \frac{l - \nu - \mu_S}{\sigma_S} \right) - \phi \left( \frac{u - \nu - \mu_S}{\sigma_S} \right) \right) \\ & + \mathbb{E} \left[ (1 - \omega - \alpha s) \phi \left( \frac{\bar{P}^1(Q^{C^*}) - \mu_S}{\sigma_S} \right) \right. \\ & \quad \left. - (1 - \alpha s) \phi \left( \frac{\bar{P}^4(Q^{C^*}) - \mu_S}{\sigma_S} \right) \middle| \tilde{\xi} \in \Omega^1 \right] \Pr(\tilde{\xi} \in \Omega^1) \\ & + \mathbb{E} \left[ (1 - \omega - \alpha s) \phi \left( \frac{P^1(Q^{C^*}) - \mu_S}{\sigma_S} \right) \right. \\ & \quad \left. - (1 - \alpha s) \phi \left( \frac{P^4(Q^{C^*}) - \mu_S}{\sigma_S} \right) \middle| \tilde{\xi} \in \Omega^6 \right] \Pr(\tilde{\xi} \in \Omega^6), \end{aligned} \tag{8}$$

where  $\phi(\cdot)$  is the pdf of the standard normal distribution. The first term in (8) captures the impact of  $\sigma_S$  on the expected marginal cost of C-input, whereas the latter terms capture the same on the expected marginal revenue. As discussed above, a higher  $\sigma_S$  increases the expected marginal cost, i.e., the first term is negative only if the window contract provides a lower upside protection than downside opportunity loss ( $\mu_S + \nu < (l + u)/2$ ).

The impact of a higher  $\sigma_S$  on the expected marginal revenue of C-input is more subtle and depends on the interplay between the spot price and product market uncertainties. To demonstrate the intuition, let us focus on a realization of  $\xi \in \Omega^1$ . In this case, the processing window at Stage 1 is characterized by  $[\bar{P}^4(Q^{C^*}), \bar{P}^1(Q^{C^*})]$ . On this sample path of  $\xi$ , the expected marginal revenue of C-input increases in  $\sigma_S$  if  $\mu_S > (\bar{P}^4(Q^{C^*}) + \bar{P}^1(Q^{C^*}))/2$  (and decreases in  $\sigma_S$  otherwise). This is because when the mean spot price is sufficiently high, with a higher  $\sigma_S$ , C-input benefits from high  $P^S$  realizations as the value of spot resale increases, whereas it is not negatively affected from low  $P^S$  realizations as much because of the processing window. Although the impact of  $\sigma_S$  on the expected marginal revenue of C-input can be characterized for each  $\xi$  realization, the overall impact in expectation with respect to  $\xi$  is ambiguous because the limits  $\bar{P}^4(Q^{C^*}), \bar{P}^1(Q^{C^*})$  of the processing window depend on the product market prices, and in turn on  $\xi$ . A stronger result can be obtained in the special case of no spot procurement access. In the absence of spot procurement, we can prove that the expected marginal revenue of C-input increases in  $\sigma_S$ . This is because the firm uses the spot market only for resale of C-input, and it does so only when the spot price is sufficiently high. A higher  $\sigma_S$  increases the probability of a higher spot price to induce spot resale, so that the expected marginal revenue of C-input increases.

In summary, when there is no access for spot procurement, the optimal contract volume increases in spot price variability if the firm uses a fixed forward contract ( $l = u$ ), or an unconstrained contract ( $l \rightarrow -\infty, u \rightarrow \infty$ ), or any contract with higher upside protection than downside opportunity loss ( $\mu_s + \nu > (l + u)/2$ ). This result is proven to hold for the general model and not only for the special case of the price-taker firm. When the firm uses a contract with lower upside protection than downside opportunity loss or if the firm has access to spot procurement, the impact of  $\sigma_s$  on the optimal contract volume is ambiguous and is determined by the interplay between spot price and product market uncertainties.

The impact of  $\sigma_s$  on the expected spot procurement at the optimal solution is characterized by its impact on expected spot procurement for a given  $Q^C$  and the change in the optimal contract volume  $Q^{C*}$ . The spot procurement at Stage 1 is linearly decreasing in the spot price when this price is in a certain window, and outside this window, it is at full plant capacity  $K - Q^{C*}$  when the spot price is lower, and zero when the spot price is higher. Similar to the impact of  $\sigma_s$  on the marginal revenue of C-input, the impact of  $\sigma_s$  on this window is ambiguous because the limits of it depend on the product market prices. In all of our numerical experiments reported in the next section, we observe that the expected spot procurement decreases (increases) when the optimal contract volume increases (decreases). However, we do not have a proof that this apparent regularity is true in general.

## 5.2. Impact of Correlation Parameter $\alpha$

We first analyze the impact of the correlation parameter  $\alpha$  on the optimal expected profit.

**PROPOSITION 5.** *The optimal expected profit of the firm,  $V^*$ , decreases in  $\alpha$ .*

The intuition here is that the firm benefits from asymmetry between the spot price and the product market price. With a lower correlation,<sup>11</sup> there will be a higher likelihood when the spot price is low (high) that the product market price will be high (low). When the spot price is low, the firm can buy S-input at a cheaper price and, after processing, can sell the two outputs at a high price. When the spot price is high, the firm optimally does not buy from the spot market (and indeed, may resell its C-input in the spot), and the lower product market price is less consequential. In short, low product market prices and high spot

price become less consequential with a lower correlation because of the increased likelihood of using available options upstream or downstream (or both).

To analyze the impact of  $\alpha$  on the optimal procurement portfolio, we will once more focus on the pure price-taker special case. The correlation parameter  $\alpha$  affects only the expected marginal revenue of C-input and not the expected marginal cost. We will first note that  $\alpha$  has an opposite impact on the marginal revenue of C-input to that associated with the impact of  $\sigma_s$ . Recall that the marginal revenue of C-input at Stage 1 is characterized by the processing option when the spot price is in a certain window; outside this window, it is characterized by the opportunity gain from not using spot procurement when the spot price is lower and spot sale revenue when the spot price is higher. On the right tail of this window, a higher  $\sigma_s$  increases the probability of a spot resale of C-input as the probability of higher spot price realizations that induce the spot resale increases. In contrast, a higher  $\alpha$  decreases the probability of a spot resale, because the processing option becomes more valuable when the spot price is high due to the increasing correlation. On the left tail of the processing window, a higher  $\sigma_s$  increases the probability of spot procurement as the probability of lower spot price realizations that induce the spot procurement increases. In contrast, a higher  $\alpha$  decreases the probability of spot procurement because the processing option with S-input becomes less valuable when the spot price is low because of increasing correlation. Therefore,  $\alpha$  and  $\sigma_s$  have opposite impacts on the expected marginal revenue of C-input.

Paralleling the argument above, it can be shown that the impact of  $\alpha$  on the optimal contract volume is of the opposite sign to the impact of  $\sigma_s$  on the expected marginal revenue of C-input, i.e., the last two terms in (8), evaluated at  $\omega = 0$ . This result also holds true for the general model and not only for the special case of the price-taker firm. Similar to the impact of spot price variability as discussed in §5.1, the impact of  $\alpha$  on the optimal contract volume is ambiguous. In the absence of spot procurement, it can be proven that  $Q^{C*}$  decreases in  $\alpha$  in the context of the general model. In this case, the expected marginal revenue of C-input is the maximum of the two available options, spot sale or processing. A lower  $\alpha$ , and thus a lower correlation, provides a natural hedge between these two options: the value of one option is higher when the other is lower. For the impact of  $\alpha$  on the expected spot procurement at the optimal solution, we can also show for the general model that this effect is of the opposite sign to the impact of  $\sigma_s$ .

## 5.3. Discussion

The managerial insights from our analysis are as follows. The firm benefits from a lower correlation

<sup>11</sup> The reader should note that an increase in  $\alpha$  increases not just correlation, but also final product price variability, so the noted effect from  $\alpha$  may be due in part to the increased variability in product prices. Moreover, as noted in the next section, computational examples show that increased product market price variability itself leads to *increased* profits, but we have no general proof of this.

between the spot price and product market uncertainties, and this is independent of the form of the window contract. The firm benefits from a higher spot price variability if the firm uses a fixed forward contract, an unconstrained contract, or any window contract with higher upside protection than downside opportunity loss. Otherwise, there exists a trade-off between a higher contract procurement cost and higher expected revenues from processing. In the absence of spot procurement, the firm should increase its contract volume with a lower correlation. The same holds with a higher spot price variability if the window contract does not have a lower upside protection than the downside opportunity loss. With spot procurement, the impact of the correlation and the spot price variability on the optimal procurement portfolio is determined by the interplay between the spot price and product market uncertainties and is ambiguous in general.

We close this section with an important remark on the value of window contract with respect to the fixed forward contract. Let  $\bar{F}$  denote the price of the forward contract, and let  $l = \bar{F} - \tau$ ,  $u = \bar{F} + \tau$  denote the parameters of the window contract that is symmetric around  $\bar{F}$ , where  $\tau < \bar{F}$ . The value of using a window contract instead of a fixed forward contract and its implications on the optimal procurement portfolio critically depend on the ordering between the forward price and the mean contract procurement price, as the next result shows.

**PROPOSITION 6.** *The optimal contract volume and the optimal expected profit are lower (higher), whereas the expected spot procurement at the optimal solution is higher (lower) with the window contract if  $\bar{F} > \mu_S + \nu$  ( $\bar{F} < \mu_S + \nu$ ).*

In the next section, we shed more light on the main drivers of the optimal procurement portfolio using numerical experiments and analytical results based on beef supply chains.

## 6. Computational Experiments for the Beef Supply Chain

This section describes computational results for the above model based on data for the U.S. beef industry described in the GIPSA (2007) report and complemented by industry demand and supply studies. The GIPSA data pertain to the period October 2002 through March 2005. We focus on an average-sized U.S. packer (see GIPSA 2007, Tables 3.2, 3.3, Figure 3.1) with a rated capacity of 25,000 heads of cattle per week (corresponding to the mean plant size of the GIPSA (2007) report of 103,733 cattle per month as reported in Table 3.2). Tables 1 and 2 provide the benchmark values for this packer and the relevant range for the sensitivity analysis we will undertake.

Contracting in the U.S. beef market is based on a “framework agreement” signed six months to a year, or longer, in advance of spot deliveries between feedlot owners and packers. Although the quality and delivery terms are specified in the “framework agreement,” quantity is not. Contracted quantity (i.e., C-input) is determined between a few weeks and a few months in advance of the spot day, and can depend on regional supplies and other factors. What is fixed in advance is the structure of the contract for C-input. The most common contract used in the U.S. fed-cattle industry is a window contract benchmarked on spot price. In terms of the general window contract specified earlier, in which contract price equals  $\max(\min(u, P^S + \nu), l)$ , the standard industry contract has no limits ( $l \rightarrow -\infty$  and  $u \rightarrow \infty$ ), and the contract adjustment parameter  $\nu$  is specified simply as  $\nu = \Delta v$ , which represents the value per unit of the quality difference  $\Delta$  per unit of C-input relative to S-input. In beef supply chains, this quality difference results from special feeding regimes undertaken for fed cattle purchased under contract (C-input) relative to the greater heterogeneity of cattle purchased in the spot market (S-input).

For computational experiments, we assume  $\tilde{\xi}' = (\tilde{\xi}_1, \tilde{\xi}_2)$  to follow a bivariate normal distribution and  $P^S$  to follow a normal distribution.<sup>12</sup> Because no information is available in the GIPSA study on the correlation between the spot price and final product prices, this correlation was assumed to be zero for these experiments, i.e.,  $\alpha = 0$  in the price equations.<sup>13</sup> We programmed the first-order condition and the other performance measures in MATLAB. We validated the code against a number of tests that included making comparisons between the MATLAB results and (i) explicitly calculated optimal values for the performance measures when  $\xi$  and  $\tilde{P}^S$  equaled their mean values (in this case,  $\sigma_\xi$  and  $\sigma_S$  were assigned very low values so that all the probability mass was located at the mean) and (ii) results of several special cases of the problem for which analytical results exist on the

<sup>12</sup> It follows from Tables 1 and 2 that the coefficients of variation are not large; hence the nonnegativity of the random variables embodied in our normality assumption is unproblematic.

<sup>13</sup> The appropriate correlation measure between final product prices and spot prices is that associated with the contracting decisions (one to two months in advance of the spot). The authors' analysis of this shows that for the past decade this correlation has been relatively low, on the order of 0.1 to 0.3, depending on how far in advance of the spot day the contract delivery quantity is agreed. If prices are averaged on a quarterly basis over a longer time period, this correlation is significantly higher, and can exceed 0.75 for U.S. beef markets. However, the relevant correlation for the problem studied here is the much lower correlation corresponding to quantity decisions in the four- to eight-week advance contract market. In particular, neglecting this correlation in the simulation studies here is not likely to have a significant effect on results.

**Table 1 Description of the Spot and Contract Market Characteristics in Numerical Studies**

Notation	Description	Benchmark value	Range
$\omega$	Transaction cost in spot sales (percentage)	4% of $P^S$	
$t$	Transaction cost in spot procurement	4% of $\mu_S$ (\$64/head)	
$\mu_S$	Mean spot price	\$1,600/head	–25% to 10% of the benchmark with 5% increments
$\sigma_S$	Spot price volatility	8% of $\mu_S$ (128)	4% to 9% of $\mu_S$ with 1% increments
$v$	Surcharge parameter for quality difference of C-input	(\$4,800/head) $\Delta v = 3.75\%$ of $\mu_S$	

**Table 2 Description of the Processing Characteristics in Numerical Studies**

Notation	Description	Benchmark value	Range
$c_1$	Utilization cost parameter	\$0.001	
$c_0$	Common processing cost parameter	\$100/head	
$\delta$	Nonuniformity cost of S-input	\$1.39/head	
$K$	Plant size	25,000 head/week	
$e$	Cross-price elasticity parameter	0.005	0 to 0.01 with 0.0025 increments
$b_1$	Own price coefficient for program beef	0.035	
$b_2$	Own price coefficient for commodity beef	0.01	
$\mu_1$	Mean demand of program beef	3,800	2% to 12% of the benchmark with 2% increments
$\mu_2$	Mean demand of commodity beef	3,000	
$\sigma_{\xi_1} = \sigma_{\xi_2} = \sigma_\xi$	Demand variability	6% of $\mu_2$ (180)	3% to 8% of $\mu_2$ with 1% increments
$\rho_\xi$	Demand correlation	0.9	0.75 to 1 with 0.05 increments
$a_1^S$	Fixed proportion of program beef with S-input processing	0.18	
$a_2^S$	Fixed proportion of commodity beef with S-input processing	0.42	
$\Delta$	Quality Difference = $a_1^C - a_1^S = a_2^S - a_2^C$	0.0125	
$s$	Total proportion of usable carcass = $a_1^C + a_2^C = a_1^S + a_2^S$	0.60	

behavior of the optimal performance measures. We note some of these analytical results below.

Our computational experiments focus on the impact of spot price uncertainty ( $\mu_S, \sigma_S$ ), product market uncertainty ( $\mu_i, \sigma_\xi, \rho_\xi$ ), and the cross-price elasticity parameter ( $e$ ) on the optimal procurement portfolio (the optimal contract volume  $Q^{C*}$ , the expected spot procurement at the optimal solution  $\mathbb{E}[Q^{S*}]$ , and the optimal portfolio ratio  $Q^{C*}/(Q^{C*} + \mathbb{E}[Q^{S*}])$  and the optimal expected profit  $V^*$  of the packer.<sup>14</sup> Table 3 summarizes the impact of these parameters over their entire range as specified in Tables 1 and 2. A detailed discussion of the model calibration and a more extensive analysis of the impact of several other parameters of interest on a broader set of performance measures are available in Boyabath et al. (2011).

Two remarks are helpful before reporting our numerical results: First, in all of our numerical experiments reported below, we observe that the firm optimally sells at most 3.7% of the C-input to the spot market on expectation. Therefore, although the firm has both the sale and procurement options on the spot market, for the particular plant and markets modeled here, only the spot procurement option has a significant value. Second, in all of our numerical

experiments, in the optimal solution, all of the probability mass  $\xi$  is located in the  $\Omega^1$  region. Therefore, we can focus only on this region in delineating the intuition behind the numerical observations. This observation also enables us to prove the sign of some of the comparative static results analytically. These results are highlighted in Table 3 with a box around the relevant cell, indicating that the specific numerical result shown is actually provable by assuming that all of the probability mass of  $\xi$  is located in the  $\Omega^1$  region. The results that can be proven without this assumption, i.e., the general window contract of \$4,

**Table 3 Impact of Parameters on the Performance Measures:**  
 $\uparrow\downarrow$  Implies That Expected Spot Procurement First Increases Then Decreases with an Increase in  $\mu_S$

An increase in	Contract volume	Expected spot procurement	Portfolio ratio	Expected profit
$\mu_S$	↓	↑↓	↓	↓
$\sigma_S$	↓	↑	↓	↑
$\mu_i$	↑	↓	↑	↑
$\sigma_\xi$	↓	↑	↓	↑
$\rho_\xi$	↓	↑	↓	↑
$e$	↓	↑	↓	↓

<sup>14</sup> The expected profit includes \$900,000 in fixed costs (including payments to owners/investors) per week.

are denoted with a “double box” around the cell. The proofs for the analytical results are relegated to §E of the online technical appendix.

We now discuss the intuition behind the results in Table 3. Sections 6.1 and 6.2 focus on the impact of spot price and product market uncertainty respectively. We investigate the impact of product and demand substitution on the key performance measures in §6.3. Our managerial insights are summarized in §6.4.

### 6.1. Effect of Spot Price Uncertainty ( $\mu_S, \sigma_S$ )

An increase in the mean spot price  $\mu_S$  decreases the optimal expected profit of the packer.<sup>15</sup> This is because both the expected C-input and S-input procurement cost increase. Because the expected spot procurement cost increases, the marginal revenue of C-input increases with a higher  $\mu_S$ . However, a higher  $\mu_S$  also increases the marginal procurement cost of C-input, and this outweighs the increase in the marginal revenue. Therefore,  $Q^{C*}$  decreases. On the impact of a higher  $\mu_S$  on the expected spot procurement, two effects work in the opposite directions: A lower  $Q^{C*}$  works to increase it, whereas a higher expected spot procurement cost works to decrease it. For sufficiently low values of  $\mu_S$ , the former effect dominates, and the expected spot procurement increases. For sufficiently large values of  $\mu_S$ , the packer does not contract any C-input, and only the latter effect exists. Thus, the expected spot procurement decreases with an increase in  $\mu_S$ . As  $Q^{C*}$  decreases, the optimal portfolio ratio decreases with an increase in  $\mu_S$ .

An increase in the spot price variability  $\sigma_S$  increases the optimal expected profit as follows from Proposition 4. The optimal contract volume decreases in  $\sigma_S$ . The packer uses the spot market almost exclusively for procurement, and it does so when the spot price is low. A higher  $\sigma_S$  means a higher probability that the spot price is sufficiently low to induce spot procurement. The larger reliance on the spot market decreases the marginal revenue of C-input, and the optimal contract volume decreases. Because  $Q^{C*}$  decreases with an increase in  $\sigma_S$ , the expected spot procurement at the optimal solution increases, and the portfolio ratio decreases.

### 6.2. Effect of Product Market Uncertainty

( $\mu_i, \sigma_\xi, \rho_\xi$ )

An increase in the mean demand parameter  $\mu_i$  increases the optimal expected profit of the packer. This is because the average product price increases, increasing also the expected revenues from processing. Because the value of processing increases,

the marginal revenue of C-input increases, and the packer increases the optimal contract volume with a higher  $\mu_i$ . Because  $Q^{C*}$  increases with an increase in  $\mu_i$ , the expected spot procurement at the optimal solution decreases, and the portfolio ratio increases.

For the effect of  $\sigma_\xi$  and  $\rho_\xi$ , we note here that  $\tilde{\xi}$  appears in the form of  $\tilde{W} = h_1\tilde{\xi}_1 + h_2\tilde{\xi}_2$  for  $h_i \in \{a_i^C, a_i^S\}$  in a linearly increasing fashion both in the marginal revenue of C-input and the expected profit of the packer for a given  $Q^C$ . Because  $\tilde{\xi}$  is bivariate normal,  $\tilde{W}$  is normally distributed with mean  $h_1\mu_1 + h_2\mu_2$  and standard deviation  $\bar{\sigma} \doteq \sigma_\xi \sqrt{h_1^2 + h_2^2 + 2h_1h_2\rho_\xi}$ . Therefore, increasing  $\sigma_\xi$  or  $\rho_\xi$  leads to a higher product market variability  $\bar{\sigma}$ . Because the firm optimally processes only when product market return is sufficiently high, a higher  $\bar{\sigma}$  increases the value of the processing option of the firm, and the optimal expected profit of the firm increases. For the impact of  $\sigma_\xi$  and  $\rho_\xi$  on the optimal contract volume, the packer uses spot procurement only when the product market return is sufficiently high (for a given spot price realization). A higher  $\bar{\sigma}$  means a higher probability that the product market return is sufficiently high to induce spot procurement. The larger reliance on spot market decreases the marginal revenue of C-input, and the optimal contract volume decreases. Because  $Q^{C*}$  decreases with an increase in  $\sigma_\xi$  or  $\rho_\xi$ , the expected spot procurement at the optimal solution increases, and the portfolio ratio decreases.

### 6.3. Effect of Demand and Product Substitution

The effect of *demand substitution* (through the cross-price elasticity parameter  $e$ ) is driven by the change in the product market profitability. As  $e$  increases, because the two outputs are substitutes, the firm is not able to price differentiate between the two markets because of the higher cross-price effect, and the profitability of the product market decreases. Therefore, a higher  $e$  decreases the value of processing, and thus the optimal expected profit of the packer. With an increase in  $e$ , because the value of processing decreases, the marginal revenue of C-input, and thus  $Q^{C*}$ , decreases. A lower dependence on contract purchases leads to a higher expected spot procurement and a lower contract ratio.

The effect of *product substitution* is driven by the product substitution regime used by the firm. From our computational experiments, we observe that product substitution does not have any value for the calibration implied by the GIPSA data: The packer optimally does not use any product substitution. This observation is consistent with empirical observations, as packers rarely convert premium product (program beef) to standard product (commodity beef) in practice. We note here that the ineffectiveness of product substitution depends partly on

<sup>15</sup> The same result is proven to hold for the general window contract for  $\alpha \geq 0$  and  $\Phi((u - v - \mu_S)/\sigma_S) - \Phi((l - v - \mu_S)/\sigma_S) \geq 1 - \omega$ , where  $\Phi(\cdot)$  is the cdf of the standard normal distribution.

the high value of  $\rho_\xi$ . As follows from Proposition 1, the optimal substitution regime is determined by the difference between two market prospects ( $\tilde{\xi}_1 - \tilde{\xi}_2$ ). As  $\rho_\xi$  decreases, the asymmetry between  $\tilde{\xi}_1$  and  $\tilde{\xi}_2$  increases, and the packer starts using partial and full product substitution regimes. We observe numerically that the expected premium product substitution ratio,  $\mathbb{E}[x_{12}^*]/(\mathbb{E}[x_{11}^* + x_{12}^*])$ , increases with a decrease in  $\rho_\xi$  for sufficiently negative correlation levels. In this case, product substitution does have a significant effect on the optimal procurement portfolio and the expected profit of the packer.

#### 6.4. Discussion

The managerial insights from our analysis are as follows. A lower mean spot price and a higher mean demand increase the packer's profitability as well as increase contract procurement relative to spot procurement. The packer also benefits from a higher spot price variability, a higher product market variability, and a higher correlation between product markets. With an increase in any three of these variability measures, the packer should decrease contract volumes and rely more on spot procurement. Higher demand substitution is detrimental to the packer's profitability and reduces dependence on contract procurement, but product substitution does not have any significant effect on the packer's decisions and performance.

These results on variability, both upstream and downstream, show the interplay between the options value of contract markets and the volatility of prices. One of the most important elements of the beef context is the fact that contract prices and spot prices are closely linked through the standard contract. Even with this close link, the sensitivity of the optimal portfolio to variability in both upstream and downstream markets is evident from Table 3. What this indicates is a strong interaction among upstream and downstream factors. These factors vary considerably over time depending on supply and demand of the respective cattle entering into these two markets (e.g., see Figure 2.1 and the ensuing discussion in the GIPSA 2007 report). As a result, what one can expect is that the optimal portfolio, and the value of the contract market itself, will change over time as determinants of supply/demand and prices change. Indeed, an important contribution of the framework developed here is providing improved understanding of how the optimal supply portfolio should change in response to varying environmental conditions in a context in which plant size and technology are fixed, and high plant utilization is fundamental to profitability.

## 7. Conclusions

The model and results here provide insights on optimal procurement decisions for a fixed proportions, multiple-product technology with uncertainties

in both input prices and output prices/demands. The central question analyzed was the structure of optimal sourcing portfolios between spot sourcing and long-term contracts, with the latter taken to be of a general window form, linear in the spot price but capped by upper and lower limits on realized contract price. Our analysis provides managerial insights, as summarized in §5.3, on the interaction of window contract terms with processing options. Specializing our generalized contract form to the standard contract in use in the beef industry, we illustrate the significant impact on profits of integrated risk management in this fixed proportions supply chain. In particular, using a calibration based on the GIPSA (2007) report, this paper elucidates for the first time the value of long-term contracting in the beef supply chain. Our comparative statics results provide some rules of thumb for the packer for the strategic management of the procurement portfolio, as summarized in §6.4.

The results of this paper underline the significant benefits of coupling input risk management (through sourcing decisions) and output risk management (through pricing, production, and product substitution decisions). This theme of integrated risk management of supply is becoming increasingly central as commoditization of intermediate product markets continues, driven by global markets and the need for standardization, and as business-to-business markets continue to develop in providing the requisite contracting and hedging instruments for integrated risk management.

Relaxing the assumptions made here on the production environment gives rise to a number of interesting areas for future research, both in the theory of multiproduct production and in specific application areas such as cocoa, oil, and soybeans where a single input gives rise to multiple outputs in fixed proportions. First, there are our simplifying assumptions of a single-period, single-contract world with only two final products. The fixed proportions technology problem in the presence of multiple suppliers is examined by Boyabatlı (2011). The single period clearly needs to be generalized to a dynamic setting, following the examples of Kouvelis et al. (2010) and Secomandi (2010), who provide some results for the case of a single input and single output. Second, we have allowed free downward substitution in production, which is reasonable in the case of beef processing, but might well entail penalty and options costs in other contexts, as analyzed by Dong et al. (2010b). Third, whereas our model encompasses fairly general contract forms, including fixed forwards and general window contracts common in many markets, in other contexts, the price of contract purchases could well include alternative options features and could be subject to other determining factors (e.g., the competitive

model developed by Wu and Kleindorfer 2005). Moreover, even for other live animal supply chains, such as pork hog and broiler chicken, there are important differences from the beef market (e.g., for the pork-hog market, one would see  $a_1 > a_2$  in contrast to the beef supply chain, and the optimal operating regime would therefore occur in different regions of the  $\xi$  space, with important consequences for substitution results). These comments and noted limitations suggest a number of open research questions.

Concerning risk management, our focus has been on physical procurement only. Extensions to overlay the cash flows from this physical problem with financial hedging are an important area of future research. In the beef industry, for example, there are significant variations over time in market conditions and operating profits of packers. To the extent that profit smoothing would avoid financial transaction costs under such variable market conditions, financial hedging can be of significant value. Financial derivatives defined on either input or output markets can serve this purpose. The existence of such hedging opportunities can also affect operational decisions, as the work of Chod et al. (2010b) and Secomandi (2010) shows.

In addition to short-term issues, there are also important capacity investment and technology choice issues in the longer term. Intuitively, it is clear that the trade-offs involved between scale economies and operational flexibility (in downward substitution and yields) are likely to be richer and more complex in a fixed proportions technology world as analyzed in the present paper than in the single-input, single-output world that has been the focus of the supply risk management literature to date.

### Acknowledgments

The authors thank Shen Xian of Singapore Management University (SMU) for her assistance in the numerical experiments. They also thank reviewers and the associate editor for helpful comments. Financial support from SMU under Project Fund C207-WSMU-003 is gratefully acknowledged by the first author, as is the financial support from the INSEAD-Wharton Alliance by the second author.

### References

Bitran, G. R., S. M. Gilbert. 1994. Co-production processes with random yields in the semiconductor industry. *Oper. Res.* 42(3) 476–491.

Boyabatli, O. 2010. Supply management in multi-product firms with fixed proportions technology. Working paper, Singapore Management University, Singapore.

Boyabatli, O., P. R. Kleindorfer, S. R. Koontz. 2011. Procurement risk management in beef supply chains. P. Kouvelis, L. Dong, O. Boyabatli, R. Li, eds. *Handbook of Integrated Risk Management in Global Supply Chains*. Wiley-Blackwell, Hoboken, NJ. Forthcoming.

Burer, S., P. C. Jones, T. J. Lowe. 2008. Coordinating the supply chain in agricultural seed industry. *Eur. J. Oper. Res.* 185(7) 354–377.

Cachon, G. 2003. Supply chain coordination with contracts. S. Graves, T. de Kok, eds. *Handbooks in Operations Research and Management Science: Supply Chain Management*. North-Holland, Amsterdam, 227–339.

Chod, J., D. Pyke, N. Rudi. 2010a. The value of flexibility in make-to-order systems: The effect of demand correlation. *Oper. Res.* 58(4) 834–848.

Chod, J., N. Rudi, J. A. Van Mieghem. 2010b. Operational flexibility and financial hedging: Complements or substitutes? *Management Sci.* 56(6) 1030–1045.

Dong, L., P. Kouvelis, P. Su. 2010a. Global facility network design with transshipment and responsive pricing. *Manufacturing Service Oper. Management* 12(2) 278–298.

Dong, L., P. Kouvelis, X. Wu. 2010b. The value of operational flexibility in uncertain supply and demand environments with oil refinery applications. Working paper, Olin School of Business, Washington University in St. Louis, St. Louis.

Geman, H. 2005. *Commodities and Commodity Derivatives*. Wiley Finance, New York.

Gerchak, Y., A. Tripathy, K. Wang. 1996. Co-production models with random functionality yields. *IIE Trans.* 28(5) 391–403.

Grain Inspection, Packers and Stockyards Administration (GIPSA). 2007. GIPSA livestock and meat marketing study, vol. 3: Fed cattle and beef industries. Report, U.S. Department of Agriculture, Washington, DC.

Hsu, A., Y. Bassok. 1999. Random yield and random demand in a production system with downward substitution. *Oper. Res.* 47(2) 277–290.

Kazaz, B. 2004. Production planning under yield and demand uncertainty with yield-dependent cost and price. *Manufacturing Service Oper. Management* 6(3) 209–224.

Kleindorfer, P. R., D. J. Wu. 2003. Integrating long-term and short-term contracting via business-to-business exchanges for capital-intensive industries. *Management Sci.* 49(11) 1597–1615.

Koontz, S. R., J. D. Lawrence. 2010. Impacts of alternative marketing agreement cattle procurement on packer costs, gross margins, and profits: Evidence from plant-level profit and loss data. *Agribusiness* 26(1) 1–24.

Kouvelis, P., R. Li, D. Qing. 2010. Managing storable commodity risks: role of inventories and financial hedges. Working paper, Singapore Management University, Singapore.

Li, C., P. Kouvelis. 1999. Flexible and risk-sharing supply contracts under price uncertainty. *Management Sci.* 45(10) 1378–1398.

Lowe, T. J., P. V. Preckel. 2004. Decision technologies for agribusiness problems: A brief review of selected literature and a call for research. *Manufacturing Service Oper. Management* 6(3) 201–208.

MacDonald, J. M. 2003. Beef and pork packing industries. *Vet. Clin. North Amer. Food Admin. Pract.* 19(2) 419–443.

Mendelson, H., T. Tunca. 2007. Strategic spot trading in supply chains. *Management Sci.* 53(5) 742–759.

Roe, B., T. L. Sporleder, B. Belleville. 2004. Hog producer preferences for marketing contract attributes. *Amer. J. Agricultural Econom.* 86(1) 115–123.

Secomandi, N. 2010. Optimal commodity trading with a capacitated storage asset. *Management Sci.* 56(3) 449–467.

Tomlin, B., Y. Wang. 2008. Pricing and operational recourse in coproduction systems. *Management Sci.* 54(3) 522–537.

U.S. Department of Agriculture. 2009. Agricultural statistics. Report, U.S. Department of Agriculture, Washington, DC.

Van Mieghem, J. A., N. Rudi. 2002. Newsvendor networks: Inventory management and capacity investment with discretionary activities. *Manufacturing Service Oper. Management* 4(4) 313–335.

Wu, D. J., P. R. Kleindorfer. 2005. Competitive options, supply contracting, and electronic markets. *Management Sci.* 51(3) 452–466.