Chapter 9
The Cautious Farmer and the Local Market

Market Participation Under Uncertainty

(Economides–Siow Problem)

In market formation, risk-averse individuals choose larger markets to reduce price or liquidity risk. In a spatial economy, larger markets mean the marginal participant incurs added shipping costs. In this sense, the scale of the market may be limited by the tradeoff between the benefits and costs of participation. Model 9A assumes shipping costs are zero—absent a cost disadvantage, the firm (a farm here) is ever better off the larger the market. In Models 9B and 9C, nonzero shipping costs may limit size of market for a marginal participant. However, what about spatial equilibrium? If some farms are close to the market and others further away, there will be an incentive for farms at greater distance to relocate nearer the market. Model 9B assumes that farms reach spatial equilibrium by forming a cooperative in which members share the aggregate cost of shipping equally. Alternatively, Model 9C assumes that farms reach spatial equilibrium by bidding up the price (rent) for land at advantageous locations. Chapter 9 is the first in this book to look at firms as both producers and consumers. As we progress toward a model of location that characterizes the regional economy, it is important to integrate demand and supply. In Chapter 7, we began to think about the nature of a firm. There is a parallel here in the contrast between Models 9B and 9C. Model 9B redefines the nature of a firm because the cooperative internalizes a market transaction (for shipping) in the same way that a firm internalizes when it does some aspect of repair production in-house. This chapter therefore looks at how another aspect of localization (market organization) and price are jointly determined.

9.1 The Economides–Siow Problem

So far, we have looked at a *market* mainly in terms of the constraint on participation posed by the existence of shipping costs. In the case of an *expendable* commodity, a customer is seen to participate in a market if the *effective price* (purchase price plus *unit transaction cost*) does not exceed the *maximum price* the customer would be willing to pay. As seen in Chapter 8, a unit shipping cost—which increases with distance shipped—limits the geographic extent of a market to its *range*. In the

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case of an *indispensable* commodity, there is no corresponding notion of a market's geographic extent.

In Chapter 8, we considered the possibility of multiple suppliers of the same product at different places. In the presence of a unit shipping cost, the *firm* is thought to have a *trade area* that it shares with its competitors. A firm's *market area*—the part of the trade area in which it dominates the competitors—can then be identified on the assumption that customers purchase from the supplier with the lowest *effective price*. However, this approach continues the assumption that a commodity is expendable without considering explicitly when, how, or why substitutes are available. Put differently, the models considered so far do not say anything about why or when someone would participate in a market for this particular product. The closest we have come so far is Model 8G where a *utility* function was introduced to model consumer choice between two commodities; in principle, this might mean that the consumer foregoes the market for one product in favor of the other. In this chapter, I begin to think about what motivates individuals to seek out a particular *local market*.

Let me now cast the problem more generally. Why do *local markets* exist at various places across the landscape? So far, we have treated the *local demand* for a product and the *local supply* as *exogenous*. To this point, we have not asked what participants might do in the absence of a particular local market. In choosing to participate in a particular local market, a participant has more to gain than simply the ability to purchase a product. Presumably, to the extent that there is more than one possible market, participants might be attracted to one market over another because they think they are more likely to get a good price (i.e., a lower effective price) there. Underlying this idea is the notion that markets are inherently uncertain. Otherwise, if market outcomes were known with certainty, why would not otherwise-identical consumers always choose the same market?

In this and the remaining chapters of this book, I look at models of agricultural markets. In each of them, I assume tenant farmers—as firms—produce a crop using rented land as an input and then ship all or part of that crop to a central place to exchange. I assume here initially that all land is equally fertile. I assume also that landlords maximize rents, are competitive (not collusive), and are numerous enough that each is a price taker in the local market for land. Such models can be used to describe a simple kind of regional economy. What makes these models interesting in part is that each firm occupies geographic space, and there is therefore a limit on the number of them that can be accommodated within a given land area.[1] In later parts of this chapter, I use that feature of the model in thinking about how large a market might be in terms of the number of market participants.

To the farmer, there are costs to exchange: e.g., the cost of shipping produce to a local market and possibly purchases back home. Presumably, a farmer plans a particular mix of produce taking into account the farm's preferences, available resources, including skills of its workforce, and fertility of soil. Regardless of its plans, the harvest reaped by the farm depends also on events such as climatic variations that

[1] I first considered density of customers in Chapter 7 and then again in Chapter 8 in more detail.

are beyond the farm's control. To maximize its utility, a farm whose harvest consists mainly of grain production might want to exchange with other farmers whose harvest was mainly other kinds of desirable produce: e.g., eggs, milk, meat, or vegetables. As a supplier, you decide the type and quantity of produce to bring to market without knowing in advance what will be brought by other farmers. As a consumer, in deciding whether to attend a local market, you cannot be sure in advance about the availability, quality, and exchange rate (price) of various produce. In the interest of simplicity, assume buyers and sellers have perfect information once they arrive at the local market and that this translates into a single Walrasian market-clearing exchange rate for the day; all those who want to exchange produce at that rate are able to do so. Under what conditions will a farm participate in a particular local market? If a farm has a choice between participating in a local market nearby or a larger market further away, which will it choose and why?

This chapter is inspired by a pioneering spatial model of liquidity and market size in Economides and Siow (1988).[2] At the same time, the E&S model is different from the one presented here. How? First, the E&S model assumes that actors maximize expected utility. Here, however, I assume behavior based on risk-return. A second difference from the E&S model is that I start with a version where geography plays no role. Later in the chapter, I introduce into this model a geography different from the E&S model. I will provide more detail on these and other differences later in this exposition.

9.2 The Barter Market

Up to this point, we have looked only at markets in a *fiat money economy* wherein each commodity is exchanged for money. In this chapter, I introduce the notion of a *barter market*[3] in which farmers meet to exchange commodities: i.e., they are both producers and consumers of produce. Put differently, buyers and sellers exchange commodities in barter rather than trade a commodity for money. Students typically think of a barter economy as primitive: something done in the absence of money. Another way to think about a barter economy is that it is essentially no different from a fiat money economy. Proponents here say that money itself is just one more

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[2]Economides and Siow (1988) are primarily concerned with the existence and size of financial markets. However, at the outset, that paper describes a simple locational model of trading by farmers, which is the focus of this chapter. Others who have made use of Economides and Siow (1988) to look at questions of location include Camacho and Persky (1990), Casella (2001), Gehrig (1998), Glazer, Gradstein, and Ranjan (2003), and Henkel, Stahl, and Walz (2000).

[3]I use barter here in the economic sense of an exchange—a trade of some amount of one good in return solely for an amount of another good with no money involved—that takes place in the context of a market. This is seen here strictly as a matter of business; I exclude here any exchange (e.g., an exchange of gifts) where the motivation is, at least in part, something else.

commodity. Among the reasons why people demand money is that it is a store of value; by a further transaction, you can readily get back most or all of what you gave up for it. Other commodities, such as gold, are also thought to be like money in the sense that they too are a store of value. If indeed money is a commodity, then in effect even exchanges in a fiat money economy are barter.

While economic actors can meet up to match their needs on a pairwise basis, it is not always clear how an exchange rate (price ratio) gets established between every pair of commodities in a barter economy. Suppose, for example, we have a simple barter market that includes only two farmers. For the sake of argument, assume Farmer X arrives with an endowment of 0.4 units of wheat (commodity 1) and 0.6 units of corn (commodity 2): and that Farmer Y has an endowment of 0.60 units of wheat and 0.40 units of corn: $q_{X1} = 0.40$, $q_{Y2} = 0.40$, $q_{Y1} = 0.60$, and $q_{X2} = 0.60$. In Fig. 9.1, I draw a diagram—called an Edgeworth Box—for this problem. There,

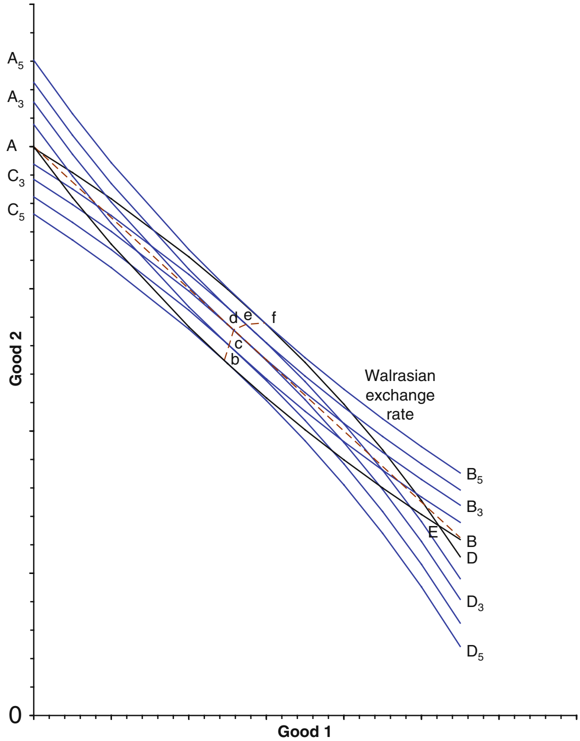

Fig. 9.1 Barter and Walrasian price setting in market with two farmers (*X* and *Y*).
Notes: $\nu = 0.3$. Initial endowments are $q_{X1} = 0.40$, $q_{X2} = 0.60$, $q_{Y1} = 0.60$, and $q_{Y2} = 0.40$. Indifference curve reached in absence of barter: AbEB for person X; AfED for person Y. Walrasian exchange rate shown as *dotted line* passing through point *A* on the *vertical axis* and point *d* along the Marshallian Contract Curve bcdef. *Horizontal axis* scaled from 0.40 to 0.75; *vertical axis* from 0.40 to 0.65

point *A* represents the endowments before any barter. Through point *A*, I have drawn Person X's indifference curve (AbEB) before barter. The idea of an Edgeworth Box is to show the indifference curves also for person Y; since total endowments are fixed, we can draw Y's indifference maps from an origin at the upper right-hand corner of Fig. 9.1. In that case, Y's indifference curve before barter would be AfED. The eye-shaped area above AbE and below AfE contains all the barter outcomes that would leave each of X and Y better off (or at least no worse off) than without barter.

Where within this eye does bartering lead us? There are two answers to this question: Marshallian and Walrasian.

Marshallian answer: Barter leads to an equilibrium in which neither party has an incentive to barter further; that is, the endowments after barter must be such that the indifference curves of X and Y are tangential at that point. In Fig. 9.1, I draw the locus of all points that satisfy this; it is labeled bcdef and generally called a Contract Curve. At point *b* on the Contract Curve, all the gains from barter accrue to Person Y, At point *f*, all the gains accrue to Person X. Points *b* and *f* are sometimes called the maximum concession points for that reason. At any point on the Contract Curve between *b* and *f*, both Person X and Person Y benefit. In the Marshallian view, where barter ends up along the Contract Curve depends on the bargaining strength of the two participants.

Walrasian answer: Barter leads to the setting of an equilibrium exchange rate between the two commodities. At that exchange rate, the two participants trade commodities until neither party has the incentive to exchange further. Therefore, we must reach the point[4] on the offer curve where the Walrasian exchange rate is tangential to the indifference curves of the two persons. In Fig. 9.1, I draw the Walrasian exchange rate as a dotted line that passes through the initial endowment (point *A*) and crosses the Contract Curve at point *d* where it is tangential to the indifference curves of both persons X and Y.[5] In the Walrasian view, barter ends up at point *d* on the Contract Curve. In moving from *a* to *d*, Person X gives up 0.07 units of corn (the amount purchased by Person Y) to purchase 0.14 units of wheat (the amount given up by Person Y).

For simplicity of exposition, I adopt the Walrasian view in the remainder of this chapter and find the exchange rate that will leave neither farmer wanting to trade any more.

[4] Implicit in this description is an assertion that such a point exists and is unique. A determination of the conditions under which this assertion is valid is beyond the scope of this book.

[5] The slope of the Walrasian exchange rate line is the negative of the exchange rate.

9.3 Uncertainty and Rationality

Economists often characterize markets as varying from *thick*[6] to *thin*[7]. As a thought experiment—with apologies in advance if the process appears mechanistic or vague—assume an asset market with the following characteristics:

- A large number of potential buyers and a large number of potential sellers. Put differently, the asset is widely desired and widely held.
- The asset is held at zero (storage) cost. However, buyers (sellers) incur transaction costs to acquire (dispose) of the asset.
- Potential buyers and potential sellers each form a statistical population of individuals. Overall, these populations are both of the view that the market price of the asset is not expected to change in the future. However, individuals within either population randomly deviate from this. On any given market day, each potential buyer (seller) has a broad sense of (1) the current price and expectations about the future price of the commodity—expectations that differ randomly from person to person and from day to day—and of (2) the transaction costs associated with their participation in the market.
- At the outset of any given market day, a subset of the potential buyers (call them market buyers) and a subset of the potential sellers (call them market sellers) engage in the market. By engage, I mean they undertake one or more of the following activities: search, gather, and analyze information; make contacts and establish relationships; negotiate price and terms; and acquire/dispose. Why only a subset? In my view, time and effort are required (i.e., transaction costs are incurred) to do these things. To acquire or dispose of a commodity, for example, these would—as noted earlier—include costs related to bank transactions and credit authorization, freight and transfer, storage and inventory, agency and brokerage fees, cost of insurance and other loss risks, installation and removal, warranty and service, and taxes and tariffs. As the day progresses, some market buyers/sellers will transact; others will—on the basis of the information obtained—choose not to transact. For simplicity, I assume that decisions to engage the market in previous days do not affect the transaction costs to be incurred today.
- Other potential buyers and sellers do not participate in the market. I assume here that the perceived transaction costs are large enough relative to the gain expected keep such potential buyers or sellers out of the market that day.
- Except insofar as transaction costs and price expectations in part vary randomly from one person to the next, market buyers and market sellers are no different

[6] A market condition in which there are many buyers and sellers. From a search-theoretic perspective on markets, a seller in a thick market does not have to wait long to get a fair price for their good.

[7] A market condition in which there are few buyers and sellers. From a search-theoretic perspective on markets, a seller in a thin market typically must wait longer to get a fair price for their goods.

statistically from other potential buyers and sellers; they can be thought of as just an independently drawn random sample.

- Market sellers in aggregate have an upward-sloping supply schedule showing the amount they would sell at any given exchange rate.
- Market buyers in aggregate have a downward-sloping demand schedule showing the amount they would purchase at any given exchange rate.
- Following the Walrasian perspective, the market exchange rate that day settles at a level such that no market seller leaves with product that they would prefer to have sold at that exchange rate and no market buyer leaves without product they would have preferred to have bought at that exchange rate. Put differently, the market clears for market buyers and sellers that day. If the market did not clear, then either market suppliers would be left with inventory or market demanders would be left with unsatisfied demands.

For an asset market that can be characterized this way, we would therefore expect to see some variation in market price from day to day even when no one expects price to change over the longer run. We also expect to see the number of market sellers or buyers rise one day and then perhaps fall the next on a random basis. I define the market to be thin when the numbers of market buyers and market sellers are small: thick when the numbers are relatively large. We might expect that the price of the asset in a thick market would be about the same from day to day because of the many participants. In a thin market under similar circumstances, however, we expect the exchange rate of a commodity to vary more from day to day. Put differently, there is more *price risk*[8] in a thin market than in a thick market; the vendor in a thin market, for example, might get less than, or more than, either potential suppliers or potential buyers think is the expected price for the asset.

Why engage in a market at all? In general, choosing a market can be seen as a means of reducing or spreading price risk. As a farmer, you do not necessarily need to participate in a weekly farmer's market. You might have, for example, established relationships with one or more customers who travel to your farm weekly to purchase commodities. Why bother with the inconvenience of shipping to market if you can get a good exchange rate at the farm gate? However, if you do not attend the market, it is hard to know whether you are getting a good exchange rate; customers too might want to know if they are paying too much. For both farmer and customer, the market provides a means of assessing whether the exchange rate for a given transaction is fair. Even a thin market can be helpful here. However, the thicker the market, the less the price risk.

[8]A loss (or increase in cost) arising because of an unforeseen change in market conditions that causes price to change over the short term, price risk is associated with price volatility. In a search-theoretic perspective, sellers hold an asset until the price bid by a potential purchaser exceeds the vendor's reservation price. Here, a distinction can be drawn between price risk and liquidity risk. Liquidity risk is the loss arising because of the delay in obtaining a bid at or above the reservation price. In practice, it is difficult to distinguish between price risk and liquidity risk. The approach in this book is to treat liquidity risk as simply an element of price risk.

To implement the notion of price risk, we need to think about what it means to be *rational* under uncertainty.[9] So far in the book, we have used the term rationality only in regard to choices made under certainty. On the one hand, we have assumed the firm maximizes profit or rate of return. On the other hand, we have assumed the individual maximizes *utility*, which in turn has been based on the consumption of commodities. In both situations, economists might argue that it is straightforward to *model* choice in the absence of uncertainty.[10]

In 1738, Daniel Bernoulli (a Swiss mathematician) developed a theory on the measurement of risk that set the stage for modern approaches.[11] He starts from the notion of an *expected value* $E[X]$. For a discrete random variable (X), this is the sum of all possible occurrences of X each multiplied by the probability, $p[X]$, of that occurrence: $E[X] = \Sigma_x x(p[X = x])$.[12] However, he then assumes that

1. the value someone places on an outcome depends not on the expected money gain from a gamble but rather on the utility it yields;
2. the added utility, $U[X]$, from a given money gain, $[X]$, is more for a pauper than for someone who is rich; and
3. the expected value of the gain in utility, $E[U] = \Sigma_x U[x]p[X = x]$, is what motivates individuals in a gamble.

Note the implication here, drawn out by Bernoulli himself (p. 29), that no one would therefore rationally gamble in a fair game—one in which a dollar gain was as likely as a dollar loss—because the loss has a greater change in utility attached to it than does the gain.

Bernoulli's assertion (2) above is problematic with respect to the ordinality of utility in two regards. First, in effect, he assumes that individuals have a *diminishing marginal utility* of income. Why might this be problematic? Diminishing marginal utility of income itself need not be surprising since we commonly assume diminishing marginal utility in commodities consumption. However, in practice, we assume that a utility function is unique up to a monotonic transformation. For example, the utility functions $f[x, y] = x^b y^{1-b}$ where $1 < b < 0$ and $g[x, y] = ax^b y^c$ where $b > 0$, $c > 0$, and $b + c < 1$, calculated at consumption of x units of wheat and y units of corn, generate the same rank ordering: i.e., $g[x, y]$ is a monotonic transformation of $f[x, y]$. The easiest way to think about diminishing marginal utility of

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[9] Important work in the area of utility and decision making under uncertainty includes von Neumann and Morgenstern (1947), Marschak (1950), Hurwicz (1953), Simon (1955, 1959), Koo (1959), Bishop (1963), Harsanyi (1965, 1966), Loomes and Sugden (1982), and Sugden (1991).

[10] For the interested reader, Sugden (1991) discusses the philosophical limitations of neoclassical perspectives on rational choice.

[11] Bernoulli (1954) is an English translation of that paper.

[12] For example, if we toss a fair coin twice and let X be the number of times a head obtains. X can take on the values 0, 1, and 2. From the Binomial Theorem, we know that probabilities are 1/4, 1/2, and 1/4, respectively. Therefore, $E(X) = 0(1/4) + (1/2)(1) + (1/4)2 = 1.0$.

income is that $b + c < 1$, but then how would this differ from a monotonic transformation of $b + c = 1$? Second, Bernoulli assumes that the utility levels of consumers (the pauper and the rich person) are comparable which again violates the ordinality of utility.

Although risk in the context of investment had long been of concern in Economics, it is Von Neumann and Morgenstern's path-breaking book, *Theory of Games and Economic Behavior*, first published in 1944, that is widely credited with spawning the focus in Economics in general on game theory and in particular on the nature of rational behavior in the presence of uncertainty.[13] That book builds on Bernoulli's idea that economic actors maximize the expected value of the gain in utility. However, there are problems with expected utility maximization. (1) In practice, how do we find the required probabilities, especially when these may be subjective (Bayesian) in nature? (2) Do individuals have a taste for risk or an aversion to risk that leads them to prefer one gamble to another even when two gambles have the same expected utility? (3) More generally, why assume that rationality necessarily requires expected utility maximization?

An alternative to analyze rational decision making under uncertainty is through a mean-variance (or, alternatively, risk-return) approach that originated with Markowitz (1952) and Sharpe (1963, 1964). Under this approach, we calculate two measures: (1) the expected utility ε—otherwise known as the mean or as the return—as per Bernoulli and (2) the variance ν—otherwise known as the risk—in utility.[14] If two choices have the same return but different risks, the individual is thought to prefer the choice with the lower risk. If two choices have the same risk but different returns, the individual is thought to prefer the alternative with the higher return. If the two choices have different returns and different risks, then we need some way to measure the tradeoff between return and risk. Typically, this is done using what is termed a *beta analysis*.[15]

In this chapter, such a risk-return approach is used to characterize rational choice under uncertainty. Here, I distinguish between sub-utility and utility. Sub-utility is the level of happiness that arises from a choice when uncertainty is, or can be, ignored. Utility is the level of happiness after uncertainty has been taken into account; in a conventional beta analysis, utility is given by (9.1.1). Beta here is a parameter that measures the aversion of the individual to risk; when $\beta = 0$, the individual is indifferent to risk, for larger β, the individual is increasingly averse to choices with substantial risk.[16]

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[13] In my view, Georgescu-Roegen (1954, p. 503) is correct in pointing out that mathematicians and statisticians dating back to Daniel Bernoulli and Gabriel Cramer had worked on similar ideas much before this. Harsanyi (1956) points out the similarities of game theory to earlier work by Zeuthen (1930).

[14] That is, $\varepsilon = \Sigma_x U[x]p[X = x]$ and $\nu = \Sigma_x (U[x] - \varepsilon)^2 p[X = x]$.

[15] Beta is the increase in mean (return) required to offset a unit increase in variance if two alternatives are to be thought to be equally preferable.

[16] This is an approach initially suggested by Markowitz (1976). See also Levy and Markowitz (1979) and Kroll, Levy, and Markowitz (1984).

Many advances have been made using a risk-return approach in an area now known broadly as financial engineering. However, the approach is not without its critics. Among these are the following:

- Some economists are not fond of the risk-return approach; they prefer an approach better grounded in neoclassical utility theory. To simplify the investment problem here, imagine an individual choosing between a risky investment with a higher expected return and a risk-free investment with a lower return. In effect, by declining the risky investment, individuals forego an amount (the amount by which the expected return is higher in the risky investment) to guarantee their wealth (the principal invested) at a future date. As such, deciding to invest in the risk-free alternative is like buying insurance and should be analyzable in that way.
- The risk-return approach does not directly incorporate an asymmetry (skewedness) of gains and losses as proposed by Bernoulli.
- The risk-return approach—as usually applied in investment analysis—assumes a continuity across investment choices: i.e., the ability to blend investments at differing levels of risk. The location problem that I consider in this chapter exhibits a kind of lumpiness that needs to be addressed specifically.

On the other hand, students tell me that they, or their parents, deal with financial advisors who regularly cast investment portfolio choices in terms of risk versus return. Therefore, I find it helpful pedagogically to cast this problem using a risk-return approach.

9.4 Model 9A: Non-spatial Market

Assume an economy made up of otherwise-identical farmers. Each farmer has the same *log-linear utility function* for sub-utility defined over the consumption of two commodities: see (9.1.2) in Table 9.1, wherein I summarize equations, assumptions, notation, and rationale for localization in Model 9A.[17] Here, let q_1 and q_2 be the amounts of wheat and corn, respectively, that the farmer consumes, and let U be the level of sub-utility achieved by the farmer. As q_1 approaches zero in (9.1.2), so does U; the same is true for q_2.

The model promotes trading by assuming that each farmer is randomly assigned—at harvest time—an initial endowment of one unit of one commodity and none of the other: i.e., (1, 0) or (0, 1). I refer to those with (1, 0) as having a wheat endowment and those with (0, 1) as having a corn endowment. In such circumstances, farmers have an incentive to exchange commodities. If they do not, their utility will be zero. An endowment here is production net of costs; for the moment, I leave unstated exactly how the agricultural commodity is produced except to say

[17] Economides and Siow (1988) also look at the case where the utility function is Constant Elasticity of Substitution (CES).

Table 9.1 Model 9A: farmers and the market in a non-spatial economy

Utility of farmer $U - \beta V$	(9.1.1)
Sub-utility of farmer $U = q_1^{\nu} q_2^{1-\nu}$	(9.1.2)
Initial endowment of farmer $P[1, 0] = 1 - \phi$ and $P[0, 1] = \phi$	(9.1.3)
Mix of farmers in market $N = N_1 + N_2$	(9.1.4)
Probability of getting N_1 wheat farmers in market $P[N_1] = C^N_{N_1}(1 - \phi)^{N_1} \phi^{N-N_1}$ where $C^N_{N1} = N!/(N_1!(N - N_1)!)$	(9.1.5)
Expected number of wheat and corn farmers in market $E[N_1] = (1 - \phi)N$ $\quad E[N_2] = \phi N$	(9.1.6)
Wheat and corn expected to be offered in market $E[Q_1] = (1 - \nu)(1 - \phi)N$ $\quad E[Q_2] = \nu\phi N$	(9.1.7)
Ratio of expected offers (wheat per unit of corn) $E[Q_1]/E[Q_2] = (1 - \nu)(1 - \phi)/(\nu\phi)$	(9.1.8)

Notes: *C*—Combination symbol: $C^b_a = b!/(a!(b - a)!)$; *P*—Probability. *Rationale for localization* (see Appendix A): Z4—Risk-spreading and insurance; Z6—Differences among consumers; Z7—Variation in goods. *Givens* (parameter or exogenous): *N*—Number of farmers in market; β—*R*isk aversion parameter; ν—Exponent of wheat in utility function. Relative preference for wheat; ϕ—*P*robability farmer has (0,1) endowment. *Outcomes* (endogenous): N_1—Number of wheat farmers in market; N_2—Number of corn farmers in market; Q_1—Aggregate quantity of wheat offered in market; q_1—Quantity of wheat consumed by farmer; Q_2—Aggregate quantity of corn offered in market; q_2—Quantity of corn consumed by farmer; *U*—Sub-utility of farmer; *V*—Variance.

that each farmer is efficient and that those who harvest corn (as well as those who harvest wheat) produce an amount of 1 unit of the commodity net of costs including the opportunity cost of land (rent). Further, assume that these initial endowments obtain as though outcomes were random and statistically independent and that for each farmer there is a probability ϕ that he or she will be endowed with corn, and therefore $1 - \phi$ probability of being endowed with wheat. See (9.1.3).

In illustrating Model 9A (and again in Model 9B and Model 9C that follow), I use particular values for ν and ϕ. I assume $\nu = 0.3$, which implies consumers prefer to consume relatively more wheat than corn. I assume $\phi = 0.4$ which means that farmers are more likely to be endowed with wheat than corn. Together, these two values describe a world in which 60% of farmers are endowed with wheat, but where each farmer wants to spend only 30% of his or her endowment on wheat consumption. In that sense, our farmers would be happier if endowments of corn were more commonplace (in other words, if $1 - \phi$ were closer to ν) and less happy otherwise. This further contributes to the imperative to trade. If they do participate in a market, they give up a portion of their initial endowment to get some amount of the other commodity to consume.

Suppose N farmers constitute a market for this purpose: see (9.1.4). To simplify the subsequent analysis, I assume that farmers decide on their size of market in advance of knowing either their endowment or that of anyone around them (i.e., the endowments of other farmers who might be in the same market). This assumption may seem strange. After all, why not let farmers choose their market later. However, this assumption will make more sense later in this chapter when we introduce geography into the model. Having decided on a size of market and having subsequently harvested and realized their endowment, I assume that the farmers then meet in this market, transact as best they can, and get the utility that arises to all farmers with their endowment at the conclusion of the market. As stated above, the endowment for each farmer among these N is stochastic and independent of the endowment of any other farmer. Then, we can think of the mix of endowments at the marketplace as the outcome of a *Bernoulli process*[18]: i.e., consists of N independent trials (one for each farmer) wherein there is a fixed probability ϕ that a particular outcome—a (0, 1) endowment—occurs.[19] In that case, the market outcomes, as measured by N_1—the number of occurrences of a wheat farmer—follow a Binomial probability distribution: see (9.1.5). The number of corn farmers in this market is $N_2 = N - N_1$ and the fraction that they make of the market is $k = N_2/N$. Because N_1 is a stochastic variable that is binomially distributed, it has a known expected value, $E[N_1]$: see (9.1.6). Because N_2 is a linear function of N_1, it too is a stochastic variable and has a binomial distribution with a calculable expected value.

In a market of size N, we expect (on average) that farmers with a wheat endowment and farmers with a corn endowment will each offer in total the amounts of wheat and corn shown in (9.1.7) in exchange for the other crop. The ratio of these—the exchange rate of corn in terms of wheat—is shown in (9.1.8).[20] However, the actual exchange rate will differ from the ratio of expected offers because k can (and often does) differ from ϕ. If we have N_1 wheat farmers and N_2 corn farmers in the market, wheat farmers will offer a total of $(1 - \nu)N_1$ units of wheat, corn farmers will offer a total of νN_2 units of corn, and on average an equilibrium exchange rate of $(1 - \nu)(1 - k)/(\nu k)$ will therefore result.

However, we do not always get this average exchange rate: as when farmers arrive at the market to discover to their chagrin that $k = 0$ or $k = 1$.[21] Consider a

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[18]A Bernoulli trial is a statistical experiment which can result in only one of two possible realizations. An experiment consisting of a series of independent Bernoulli trials is called a Bernoulli process.

[19]We would *not* have a Bernoulli process if each individual could wait until harvest time to see his endowment and that of his or her neighbors before deciding in which local market to participate.

[20]Economists usually say in this case that wheat is *numéraire* which means that other goods (corn in this case) are valued in units of wheat.

[21]In an earlier footnote, I raised the question of whether a Walrasian outcome existed and was unique. In the case of a log-linear utility function, the answer intuitively is straightforward. Each farmer maximizes utility by allocating income so that the proportions spent on wheat and corn are α and $1 - a$, respectively. At the equilibrium exchange rate, $(1 - \alpha)(1 - k)/(ak)$, a Walrasian solution exist; the market clears and farmers of each endowment are as well off as possible. The Walrasian solution is also unique; no other exchange rate clears the market.

Table 9.2 Possible realizations in market of size 2 wherein $\nu = 0.3$ and $\phi = 0.4$

				Wheat endowment			Corn endowment				
N_2 [1]	N_1 [2]	k [3]	$P[k]$ [4]	q_1 [5]	q_2 [6]	U_1 [7]	q_1 [8]	q_2 [9]	U_2 [10]	$W[k]$ [11]	Ex [12]
(a) Model 9A (Non-spatial)											
0	2	0.00	0.36	1.00	0.00	0.00	–	–	–	0.000	–
1	1	0.50	0.48	0.30	0.30	0.30	0.70	0.70	0.70	0.500	2.33
2	0	1.00	0.16	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.240	2.33
Variance										0.082	
(b) Model 9B (Spatial: g = 30, r = 0, s = 0.40)											
0	2	0.00	0.36	1.00	0.00	0.00	–	–	–	0.000	–
1	1	0.50	0.48	0.29	0.29	0.29	0.67	0.67	0.67	0.480	2.33
2	0	1.00	0.16	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.230	2.33
Variance										0.075	
(c) Model 9C (Spatial: g = 30, s = 0.40)											
0	2	0.00	0.36	1.00	0.00	0.00	–	–	–	0.000	–
1	1	0.50	0.48	0.28	0.28	0.28	0.66	0.66	0.66	0.475	2.33
2	0	1.00	0.16	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.228	2.33
Variance										0.074	

Notes: Calculations by author. See also Table 9.1. In panel (b), mean unit shipping cost is 0.0689.
–, Indicates no one of that type present. See also Table 9.1.
Ex, Exchange rate: units of wheat per unit of corn
Endogenous: $U[N]$, Utility of being in market of size N; $V[N]$ Variance in utility in market of size N; W, Average utility weighted by number of farmers of each type.

market of just two farmers ($N = 2$). Remember here that I assume farmers choose a market in advance of knowing their endowment. There are three possible realizations for N_1: 0, 1, or 2. When $\phi = 0.4$ and $\nu = 0.3$, the ratio of expected offers (9.1.8) is 3.50. For each possible outcome of N_1, the corresponding binomial probability is shown in panel (a) of Table 9.2. In the event, $N_1 = 0$ or $N_1 = 2$ here, the market consists entirely of corn endowments or wheat endowments, respectively, and the utility for each farmer in the market is therefore zero. When $N = 2$, there is a probability of 0.52[22] that the market participants will have a zero utility: i.e., return home without any of the other commodity. Suppose instead $N_1 = 1$, which means we have one farmer of each type in the market. The farmer with a wheat endowment consumes 30% of his or her endowment of wheat, and trades the remaining 70% for corn. The farmer with a corn endowment consumes 70% and trades the remaining 30% for wheat. There are two possible utilities. With a probability $(1 - k)$, the farmer will have an endowment of (1, 0), a consumption bundle

[22] 0.36 + 0.16.

(0.3, 0.3), and a utility (U_1) of 0.3. With a probability k, the farmer will have an endowment of (0, 1), a consumption bundle (0.7, 0.7), and a utility (U_2) of 0.7. In this situation, the farmer with the corn endowment is better off after trade than the farmer with the wheat endowment: not surprising given that the corn is the commodity more strongly preferred. Put differently, the farmer is here guaranteed a utility of 0.3 with a 50% chance of getting 0.7 instead. By calculating $W[k] = 0.5$,[23] I am simply taking an average of the two possible utilities weighted by the probabilities of the two outcomes. Considered over all the possible realizations of k, the weighted average utility of being in a market of $N = 2$, $U[N]$, from columns [4] and [11] in panel (a) of Table 9.2, is 0.240.[24,25] The variance in this utility, $V[N]$, is also now calculable; $V[N] = 0.082$.[26]

What is the expected exchange rate across the three possible realizations of N_1 from 0 to 2? We cannot calculate an exchange rate when $k = 0$ or $k = 1$ because no trade happens. In column [4] of panel (a) in Table 9.2, we see that the probability that $k = 0$ or $k = 1$ is 0.52. From column [12], we see that at the only other possibility, $k = 0.50$, the exchange rate is 2.33 units of wheat per unit of corn. I label this the Conditional Expected Exchange Rate (CEER); that is, the exchange rate we expect on average on the condition that k is neither 0 nor 1. This is different from the ratio of offers expected, (9.1.8), which incorporates the amounts offered when $k = 0$ and $k = 1$. In comparison, the ratio of offers expected is 3.50 when $\nu = 0.3$ and $\phi = 0.4$ as noted above.

To begin thinking about what might happen if N were larger than 2, let us do similar calculations for a market of $N = 8$ participants. As before, I continue to illustrate using the case where $\nu = 0.3$ and $\phi = 0.4$. See panel (a) of Table 9.3. Consider first the case where $N_1 = 3$. The three farmers with a wheat endowment each offer 0.7 units of wheat in exchange for corn. The remaining five farmers have a corn endowment; each offers 0.3 units of corn in exchange for wheat. The equilibrium exchange rate here is therefore 1.40[27] units of wheat per unit of corn. Each farmer with a wheat endowment therefore consumes 0.3 units of wheat, and 0.5[28] units of corn for a utility of 0.43. Each farmer with a corn endowment consumes 0.7 units of corn and trades away the remaining 0.3 units in exchange for 0.42[29] units of wheat to achieve a utility of 0.60. Therefore, the weighted average of utilities, $W[k]$, is now 0.536.[30] Considered over all the possible realizations of k as shown in Table 9.3, the weighted average utility of being in a market of $N = 8$, $U[N]$, from columns [4] and [11] in panel (a) of Table 9.3, is 0.423 and the variance $V[N]$ is now 0.087.

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[23] $0.5(0.3) + 0.5(0.7)$.

[24] $0.52(0.00) + 0.48(0.50)$.

[25] This is another place where ordinalists might well cringe. If utility is indeed ordinal, what does it mean to take a linear combination of utilities as we do when we calculated $U[N]$.

[26] $0.52(0.00 - 0.24)^2 + 0.48(0.5(0.30 - 0.24)^2 + 0.5(0.70 - 0.24)^2)$.

[27] $(3(0.7)/(5(0.3))$.

[28] $0.7/1.40$.

[29] $(0.3)(1.40)$.

[30] $(3/8)(0.43) + (5/8)(0.60)$.

Table 9.3 Possible realizations in market of size 8 wherein $\nu = 0.3$ and $\phi = 0.4$

				Wheat endowment			Corn endowment				
N_2 [1]	N_1 [2]	k [3]	$P[k]$ [4]	q_1 [5]	q_2 [6]	U_1 [7]	q_1 [8]	q_2 [9]	U_2 [10]	$W[k]$ [11]	Ex [12]
(a) Model 9A (Non-spatial)											
0	8	0.00	0.02	1.00	0.00	0.00	–	–	–	0.000	–
1	7	0.13	0.09	0.30	0.04	0.08	4.90	0.70	1.25	0.224	16.33
2	6	0.25	0.21	0.30	0.10	0.14	2.10	0.70	0.97	0.348	7.00
3	5	0.38	0.28	0.30	0.18	0.21	1.17	0.70	0.82	0.437	3.89
4	4	0.50	0.23	0.30	0.30	0.30	0.70	0.70	0.70	0.500	2.33
5	3	0.63	0.12	0.30	0.50	0.43	0.42	0.70	0.60	0.536	1.40
6	2	0.75	0.04	0.30	0.90	0.65	0.23	0.70	0.50	0.539	0.78
7	1	0.88	0.01	0.30	2.10	1.17	0.10	0.70	0.39	0.488	0.33
8	0	1.00	0.00	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.423	4.84
Variance										0.087	
(b) Model 9B (Spatial: g = 30, r = 0, and s = 0.40)											
0	8	0.00	0.02	1.00	0.00	0.00	–	–	–	0.000	–
1	7	0.13	0.09	0.28	0.04	0.07	4.52	0.65	1.16	0.207	16.33
2	6	0.25	0.21	0.28	0.09	0.13	1.94	0.65	0.90	0.320	7.00
3	5	0.38	0.28	0.28	0.17	0.19	1.08	0.65	0.75	0.403	3.89
4	4	0.50	0.23	0.28	0.28	0.28	0.65	0.65	0.65	0.461	2.33
5	3	0.63	0.12	0.28	0.46	0.40	0.39	0.65	0.55	0.494	1.40
6	2	0.75	0.04	0.28	0.83	0.60	0.22	0.65	0.46	0.497	0.78
7	1	0.88	0.01	0.26	1.94	1.08	0.09	0.65	0.36	0.450	0.33
8	0	1.00	0.00	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.390	4.84
Variance										0.074	
(c) Model 9C (Spatial: g = 30, and s = 0.40)											
0	8	0.00	0.02	1.00	0.00	0.00	–	–	–	0.000	–
1	7	0.13	0.09	0.27	0.04	0.07	4.35	0.62	1.11	0.199	16.33
2	6	0.25	0.21	0.27	0.09	0.12	1.86	0.62	0.86	0.308	7.00
3	5	0.38	0.28	0.27	0.16	0.19	1.04	0.62	0.72	0.388	3.89
4	4	0.50	0.23	0.27	0.27	0.27	0.62	0.62	0.62	0.444	2.33
5	3	0.63	0.12	0.27	0.44	0.38	0.37	0.62	0.53	0.476	1.40
6	2	0.75	0.04	0.27	0.80	0.57	0.21	0.62	0.45	0.479	0.78
7	1	0.88	0.01	0.27	1.86	1.04	0.09	0.62	0.35	0.433	0.33
8	0	1.00	0.00	–	–	–	0.00	1.00	0.00	0.000	–
Expected value										0.375	4.84
Variance										0.069	

Note: Calculations by author. See also Table 9.2. In panel (b), mean unit shipping cost is 0.12.

What is CEER when $N = 8$? Here we see from column [4] in panel (a) of Table 9.3 that the probability that $k = 0$ or $k = 1$ is 0.018, and from column [12] that, for k in between, the exchange rate varies from 16.33 down to 0.33 which yields CEER = 4.84. This is higher than the 3.50 we noted above for the ratio of expected offers: the opposite of what we had found when $N = 2$.

In this model, the farmer chooses the size of market in which to trade. This means the farmer will compare the combination of return, $U[N]$, and risk, $V[N]$, with those achievable at other sizes of market.[31] We are now able to compare the case of $N = 2$ and $N = 8$. When $N = 8$, the return is larger (0.423 vs. 0.240) but so too is the risk (0.087 vs. 0.082) compared to $N = 2$. In a risk-return analysis, the farmer would therefore prefer $N = 8$ over $N = 2$ if β is sufficiently small and prefer $N = 2$ otherwise.

To understand what is happening to CEER, suppose we let N vary from 2 to 30 and calculate CEER at each N. The resulting estimates of CEER are plotted in Fig. 9.2. The dotted line is the ratio of expected offers (9.1.8); it is horizontal because this ratio is the same at every N.[32] The solid curve shows CEER as a function of N. CEER is below the ratio of expected offers when $N = 2$, rises quickly, and peaks well above the ratio of expected offers at about $N = 8$, then begins to fall off asymptotically to the ratio of expected offers as N becomes large. We are now ready to answer some questions.

- Why is the equilibrium exchange rate low at small N? This is because (1) the exchange rate is a declining function of k, (2) the equilibrium exchange rate cannot be calculated at $k = 0$ or $k = 1$, and (3) $P(k = 0)$ is larger than $P(k = 1)$

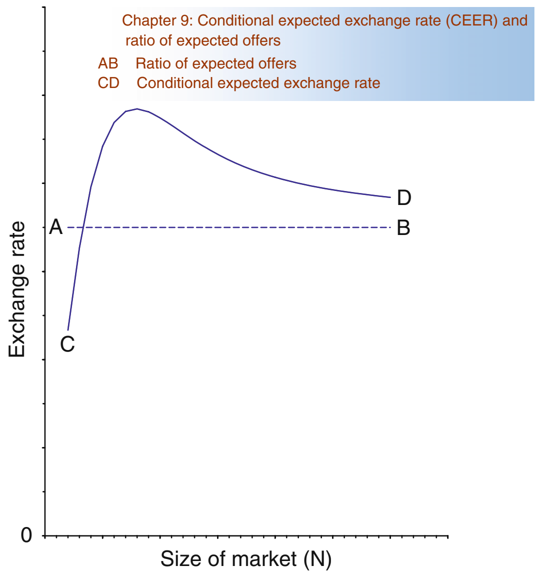

Fig. 9.2 Conditional expected exchange rate (CEER) and size of market. *Notes*: $\alpha = 0.30$ and $\phi = 0.40$. *Horizontal axis* scaled from 0 to 30; *vertical axis* from 0 to 6

[31] In Economics, utility is thought to be an index (or ordering) of preference among choice. As such, utility is an ordinal measure. However, when we calculate $U[N]$, we appear to treat utility as though it were a cardinal measure. For a discussion of the issues raised, see Ellsberg (1954).

[32] In this regard, Economides and Siow (1988, p. 110) appear to err in arguing that equilibrium price is independent of N because aggregate supply and demand for each commodity are proportional to N. My sense is that this confuses CEER and the ratio of expected offers.

- Why is CEER then above the ratio of expected offers for N sufficiently large (above $N = 4$ in Fig. 9.2)? This happens because of the asymmetry of an exchange rate. To see this, suppose the quantities of the two commodities offered in the market are identical; the exchange rate here is 1.00. Now consider increasing the quantity of either the numerator or denominator. As we increase the denominator quantity, the exchange rate can drop from 1 to as low as 0. As we increase the numerator quantity, the exchange rate rises from 1 without limit. This asymmetry means that, ignoring the effect of the exclusion of $k = 0$ and $k = 1$ offers, CEER should be systematically higher than the ratio of offers expected in (9.1.8). However, this bias dissipates as the size of market becomes larger.

Of course, CEER is not the key variable here. To understand how farmers choose markets, we must look at how market size affects utility. In Fig. 9.3, I use a solid line—the risk-return curve—to connect combinations of $U[N]$ and $V[N]$ attainable—for each level of N, again from $N = 2$ to $N = 30$. The attainable combination at each N is a black dot on this curve. Further, I have labeled the size of market at selected dots in Fig. 9.3. Here, we see that, as N is increased, the mean increases and the variance starts dropping above $N = 4$. In fact, the locus of points on this curve from $N = 4$ through $N = 30$ suggests that $U[N]$ will continue to increase, and $V[N]$ will decrease albeit both ever more slowly as N becomes still larger. In a beta analysis, we assume that the farmer is willing to accept a higher risk associated with a higher reward. One such tradeoff curve is shown as a dotted line in Fig. 9.3. We can imagine a family of such dotted lines, all parallel, such that the farmer is

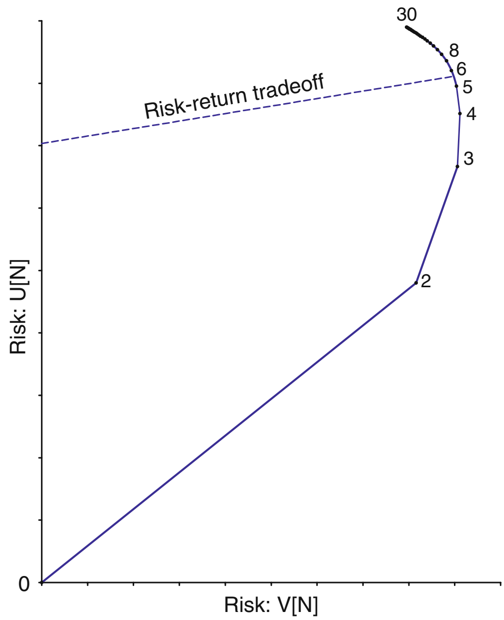

Fig. 9.3 Risk, return, and size of market.
Notes: $\alpha = 0.30$ and $\phi = 0.40$. Market size, N, shown as *labeled dots* for selected N from 2 to 30. *Horizontal axis* scaled from 0 to 0.1; *vertical axis* from 0 to 0.45

happier the higher and to the left the tradeoff curve lies. Given the risk-return curve is negatively sloped above $N = 4$, the farmer would prefer to be in as large a market as possible. This is not surprising. After all, there is no disincentive here to join a larger market, and the larger the market the higher $U[N]$ and the lower $V[N]$.

In Fig. 9.3, I also include the case of *autarky* ($N = 1$) wherein farmers do not assemble into a market. In autarky under either endowment, the farmer gets a sub-utility of 0 with certainty: i.e., a zero variance. Autarky corresponds to the origin—that is, the intersection of the horizontal and vertical axes—in Fig. 9.3 since $U[N] = 0$ and $V[N] = 0$ there.[33] If we were to draw two risk-return tradeoff lines, each parallel to the dotted line in Fig. 9.3, one drawn through the point where $N = 2$ and the other through the autarky point (the origin), we see that the utility of being in a market of size 2 exceeds the utility of not being in autarky. A similar analysis would lead us to conclude that utility is higher at $N = 3$ and still higher at $N = 4$. Put another way, the slope of the risk-return tradeoff drawn in Fig. 9.3 is sufficiently small (i.e., the risk premium, β, was low enough) to initiate an agglomeration process.

Does the reverse argument hold true? Is there a β sufficiently large that farms might never switch from isolation ($N = 1$) to a market where $N = 2$, or from there to move to markets of $N = 3$ or $N = 4$? Consider the line in Fig. 9.3 joining autarky to $N = 2$. If β were sufficiently large to make the risk-return tradeoff line steeper than this, the utility of autarky would appear to be greater than the utility of $N = 2$. Suppose alternatively the risk-return tradeoff line passing through the origin cuts the line segment joining $N = 2$ and $N = 3$. Then the utility of being in a market of $N = 3$ would be greater than the utility of autarky. If so, does that imply that farms would never get to a market size of 3, even though it is advantageous, because no one would first have the incentive to form a market of 2? If so, this would be a disturbing feature of the model because it would imply that planners would have to do what a market could not: i.e., push myopic farmers, unable to see the benefits from further agglomeration, from autarky into a market of $N = 2$ to make it possible for farmers to then form a market of 3 or more.

However, I think the problem here, specifically any reluctance to move from autarky to $N = 2$, when β is large, is actually indicative of a limitation of risk-return analysis. In autarky, the farmer receives a sub-utility of zero with certainty. When $N = 2$, the farmer receives a sub-utility of zero if both farmers have the same endowment, and a larger sub-utility if they do not. In other words, at the worst, the farmer in an $N = 2$ market does as well as in autarky. In my view, the farmer in an $N = 2$ market is always therefore at least as well off as in autarky and has the possibility of being better off. Even if we assume that the farmer is myopic about the prospect of more farmers joining the agglomeration and pushing utility even higher, there is an incentive here to form a market of $N = 2$.[34]

[33] CEER is not calculable because there is no trade.

[34] In my view, the shortcoming of risk-return analysis here is that the notion of variance loses generality when the number of realizations of a random variable (here, realizations of k) is small.

9.5 Model 9B: Cooperation in a Spatial Market

Up until now, we have assumed a non-spatial market. How does space affect this model? In earlier chapters, I have shown how shipping costs shape market size. Let us now consider how shipping costs would affect the behavior of the bartering farmers here.

In Economides and Siow (1988), farms are spread out (at fixed density) along a line left and right of the market point. As I understand the E&S model, this implies that farmers close to the market are better off than farmers who travel a longer distance to get to the market. To establish and maintain farmers in their locations in equilibrium, no farmer should be able to benefit by relocating. There must be some process to ensure that farmers are in equilibrium and do not have an incentive to further relocate. As I understand the E&S model, no such equilibrium process is specified. To correct this, I first add a cooperative process in this section (Model 9B) wherein farmers in a market form a club to share shipping costs in the market equally. In the next section (Model 9C), I introduce an alternative: a noncooperative process wherein farmers nearer the market bid a market rent premium for their plots of land compared to more remote farms. As it turns out, specifying such processes also helps us better rationalize the idea that farmers choose markets in advance of knowing their endowments. I will return to this subject shortly.

In this section, I model farms forming a club or cooperative to share the cost of participating in the market (i.e., shipping costs).[35] See Table 9.4. As used here, a club is an association whose purpose is to provide a benefit to its members. In return for paying a membership fee (f) each harvest, the club here provides free shipping services to the member. A club is like a firm except that it is not intended to make a profit: see (9.4.6).[36] Usually, there is an optimal size of club—that is to say, a desired (in this case, most efficient) membership level. As well, implementation of a club also usually involves restrictions on nonmembers who might otherwise get a free ride—that is, benefit from the activities of the club without paying the membership fee. To keep the model simple, I assume that there are no costs to the formation or enforcement of a club other than the cost of shipping. I imagine here that each farmer envisages an optimal size of club and costlessly seeks out peers with a similar view until the optimal number of members has been obtained. If, say, many farmers see an optimal club size of 10 farmers, then additional clubs will form,

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[35] See McGuire (1972) on economic models of club formation. For other modeling of cooperation in a geographic context, see Jayet (1997) and Soubeyran and Weber (2002).

[36] Here I implicitly assume contingent shipping rates. That is, the cost of shipping wheat a kilometer is s units of wheat, and the cost of shipping corn a kilometer is s units of corn. Similarly, the coop fee is contingent; it is f units of wheat if the farmer has a wheat endowment, and f units of corn if a corn endowment. There is no adjustment here for the exchange rate between wheat and corn that will be obtained in the market. For the storyteller, the advantage of this scheme is that it simplifies decision making for the farmer who is still in anticipation of the harvest and does not yet know his or her endowment.

Table 9.4 Model 9B: cooperative farmers in a spatial economy

Outer boundary of farm at ring i from market $x_i = \sqrt{(i/(\pi g))}\text{for}1 \leq i \leq N$	(9.4.1)
Mid radius for farm at ring i $m_i = \sqrt{((i - 0.5)/(\pi g))}\text{for}1 \leq i \leq N$	(9.4.2)
Endowment for farm at ring i net of shipping cost $(1 - f, 0)$ or $(0, 1 - f)$ for $1 \leq i \leq N$	(9.4.3)
Consumption of wheat and corn and sub-utility of farm with wheat endowment $q_{11} = \nu(1 - f)$ $q_{12} = (1 - \nu))(1 - f)/P$ $U_1 = (1 - f)\nu^{\nu}(1 - \nu)^{(1-\nu)}/P^{1-\nu}$	(9.4.4)
Consumption of wheat and corn and sub-utility of farm with corn endowment $q_{21} = \nu(1 - f)P$ $q_{22} = (1 - \nu)(1 - f)$ $U_2 = (1 - f)\nu^{\nu}(1 - \nu)^{(1-\nu)}P^{\nu}$	(9.4.5)
Coop's balanced budget $fN = \Sigma_i sm_i$	(9.4.6)
Minimum β for farmers participating in this co-operative $\beta \geq (U[N - 1] - U[N])/(V[N - 1] - V[N])$	(9.4.7)
Maximum β for farmers participating in this co-operative $\beta \leq (U[N] - U[N + 1])/(V[N] - V[N + 1])$	(9.4.8)

Notes: See also (9.1.1) through (9.1.6) I and Table 9.1. *Rationale for localization* (see Appendix A): Z4—Risk spreading and insurance; Z6—Differences among consumers; Z7—Variation in goods; Z8—Limitation of shipping cost. *Givens* (parameter or exogenous): *g*—*D*ensity of farms (farms per square kilometer); *N*—Number of farmers in market; *r*—Opportunity cost of land (assumed zero); *s*—Unit shipping rate; β—*R*isk aversion parameter; ν—Exponent of wheat in utility function. Relative preference for wheat; ϕ—*P*robability farmer has (0,1) endowment. *Outcomes* (endogenous): *f*—*C*o-operative fee; m_i—Mid-radius of farm *i; P*—Market exchange rate; Q_1—Aggregate quantity of wheat offered in market; q_{1i}—Quantity of wheat consumed by farmer of type *i*; Q_2—Aggregate quantity of corn offered in market; q_{2i}—Quantity of corn consumed by farmer of type *i*; *U*—Sub-utility of farmer; *V*—Variance; x_i—Outer boundary of farm *i*.

each containing 10 farmers. Market formation here is an *externality*. The action of one farmer in choosing to join a particular market affects the well-being of others in a way that is unpriced. The club, as an organizational form, is a mechanism that captures (internalizes) this externality or *spillover*.

Once formed, I assume that the club requires its members to locate so as to minimize the total shipping cost to be shared. Basically, the farms are tightly packed around the market point; this also helps control the free rider problem by keeping nonmembers further away from the market point. In what I characterize as an accretion process, assume that member farms are therefore required to form concentric rings around the market point. See the three panels in Fig. 9.4 where I map the locations of farms, each farm with the same area, when $N = 6$, $N = 4$, and $N = 2$, respectively. The farmer can be thought to ship from the mid-radius of the

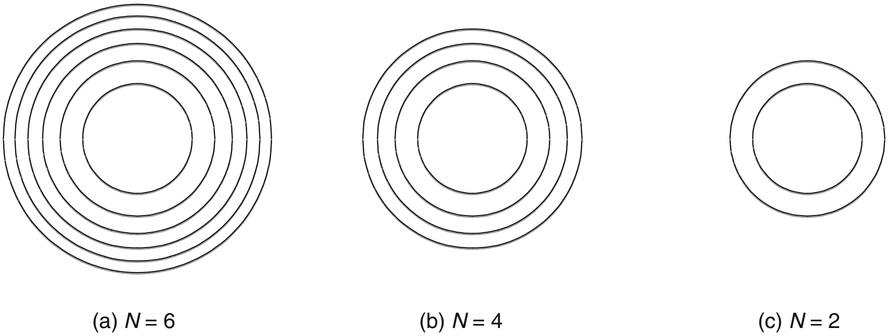

Fig. 9.4 Models 9B and 9C: Maps of farms as rings in market of $N = 6$, $N = 4$, and $N = 2$

farm.[37] The rationale for making each farm annular is that this shape minimizes the cost of shipping their endowments to the market. I ignore here other considerations that might shape the farm in a geographic sense.[38] In effect, the market is at the center of the innermost circle: the center of farm 1. Since each farm occupies the same amount of land, the shipping cost associated with the marginal farmer (i.e., the farmer furthest from the market) and the fee (average cost) paid by each farmer in a coop increases at a decreasing rate with the number of farmers in the market. See Fig. 9.5. As I have already assumed land is plentiful, I do not need to worry about the possibility of clubs with overlapping market areas; a club would simply move to an unoccupied area so that it can achieve the same low total shipping cost as any other club of the same size.

In this model, farmers may differ from one another in the following ways: (1) a randomly determined endowment not known in advance of club formation; (2) a given tradeoff between risk and return; and (3) a location vis-à-vis the market that the farmer can choose. Otherwise, I assume farmers are identical: same preferences for wheat and corn and same fee to join a given coop. Therefore, a club will be formed by farmers with similar tradeoffs between risk and return: i.e., similar βs. Once in a club, the farmer is indifferent to location because the club pays the marginal cost of shipping from that site to the market.

Assume each farmer uses a fixed amount of land, $1/g$, in agricultural production.[39] Assume that land is not used for any other purpose (we ignore here any need for land for transportation, for a market site, for housing, or for the production of

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[37] Mid-radius here is the distance which divides the farm into two equal areas.

[38] For instance, while a ring might be the most efficient shape for getting the agricultural commodity to market, it may be inefficient for the daily chores of the farmer throughout the growing season. Thünen (1966, chaps. 11 and 13) discusses aspects of this problem.

[39] This might be because each farmer has a Leontief technology that requires all inputs be in fixed proportion; however, the model to this stage is silent on other inputs to production.

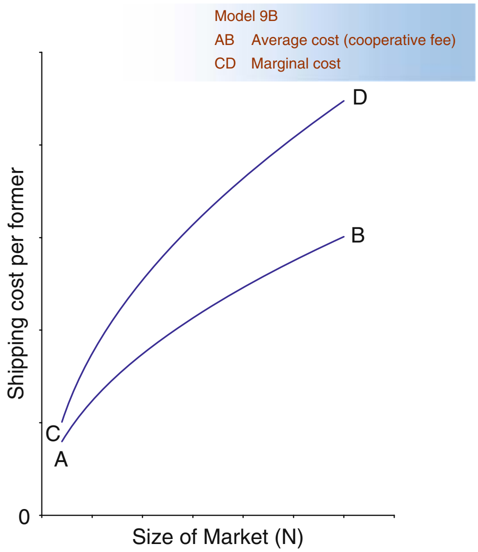

Fig. 9.5 Model 9B: Shipping cost for marginal farmer and average shipping cost (coop fee) as function of the number of farmers in the market (*N*) when farms arranged as concentric rings around market.
Notes: $g = 30$ and $s = 0.40$. Graph shown for $2 \leq N < 30$. *Horizontal axis* scaled from 0 to 35; *vertical axis* from 0 to 0.25

any other commodity). On a two-dimensional plane, we can therefore assume farmers spread at a fixed density: g farmers per unit area.[40] If N is the number of farmers participating in a market, then the outer radius of the market (i.e., the marginal farm) is $X = (N/(\pi g))^{0.5}$.

Assume the coop incurs a constant *unit shipping rate* for each of its member farmers. Therefore, for the marginal farmer at distance x from the market, the cost of shipping to the market incurred by the coop is sx, where s is the unit shipping rate per kilometer shipped (assumed to be the same for both commodities). In the empirical modeling that follows, suppose $g = 30$ and $s = 0.40$. The first farm (area of $1/g = 0.033$) stretches from an inner radius of 0 to an outer radius 0.1030; its mid-radius is 0.0728 and the shipping cost incurred by the coop for it is 0.0291.[41] The second farm (also an area of $1/g = 0.033$) stretches from radius 0.1030 to radius 0.1457; its mid-radius is 0.1262 and the marginal shipping cost for the coop is therefore 0.0505.[42] See (9.4.1) and (9.4.2).

I treat the membership fee (measured in units of the endowment and paid at the time of the harvest) as a constant per farmer.[43] For the coop, this fee is simply cost

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[40] Economides and Siow (1988) model the case where farms are spread out along a line in one-dimensional space.

[41] (0.40)(0.0728).

[42] (0.40)(0.1262).

[43] Since each farmer has an endowment of either (1, 0) or (0, 1), if I assume that each farmer carries his or her entire endowment to the market to trade (not unreasonable given that the farmer does not know the exchange rate that might be established), the shipping cost associated with each farmer is fixed whether measured per unit shipped or per farmer.

recovery: i.e., total shipping cost incurred by the coop is split equally among the N farmers in the coop: see (9.4.6).

A feature of the log-linear utility function is that consumption has a *linear expansion path*.[44] Even though the farmer's income available for consumption of wheat and corn is now net of the membership fee, consumption changes proportionally with income; the marginal farmer still spends the same proportion (ν) of his or her net income on wheat and the remainder on corn.

Here in Model 9B, I assume farmers cooperate by sharing equally the total shipping costs of all farmers in the market. If market size (N) is just 2 farms, the co-op fee (f) borne by each farmer would be 0.0398.[45] Suppose $N_1 = 1$. One farmer has an initial endowment (income net of production cost and land rent) of (1, 0). Since $\nu = 0.3$, this person prefers to consume 30% of his or her endowment net of coop fee in wheat itself, and trade the remaining 70% away for corn. See panel (b) of Table 9.2. The other farmer has an initial endowment of (0, 1) of which he or she prefers to consume 70% (again net of coop fee) and trade away the remaining 30% for wheat. See (9.4.3), (9.4.4), and (9.4.5). Since each farmer pays a coop fee equal to the average shipping cost, that exchange ratio between wheat and corn in this market is 2.33[46] units of wheat per unit of corn just as in Model 9A. If the $N = 2$ market, there are two possible utilities when $k = 0.5$: a (1, 0) endowment that yields a consumption bundle (0.29, 0.29) and a utility (U_1) of 0.29 (compared to 0.30 in Model 9A) with a probability (k) of 0.5 or a consumption bundle (0.67, 0.67) and a utility (U_2) of 0.67 (compared with 0.70 in Model 9A) with a probability $(1 - k)$ of 0.5. Then, $W[k] = 0.480$[47] (compared to 0.50 in Model 9A). The introduction of shipping cost here leaves the exchange rate unchanged but reduces the sub-utility in every outcome, not surprisingly given that the membership fee reduces the amount of wheat and corn available for consumption. As a result, $U[N]$, from columns [4] and [11] in panel (b) of Table 9.2 is 0.230,[48] down from 0.240 in Model 9A. Further, the variance in this utility, $V[N]$, is also now calculable; $V[N] = 0.075$[49] down from 0.082 in Model 9A. To conclude, in the case of $N = 2$, the introduction of shipping costs reduces both return and risk compared to Model 9A.

Now let us do similar calculations for a market of $N = 8$ participants: see panel (b) of Table 9.3. CEER remains the same as in Model 9A. However, compared to Model 9A in panel (a), we find—as when $N = 2$—that U_1, U_2, and $W[k]$ drop for any $0 < k < 1$. As a result, $U[N] = 0.390$ is smaller than for Model 9A. $V[N]$ too

[44]As used here, a condition of the utility function wherein, if as income is increased by a fixed proportion holding prices of commodities constant, the rational consumer purchases the same proportion more of each good. Put differently, each good has an income elasticity of +1.0. Such a utility function is also said to exhibit homotheticity.

[45](0.0505 + 0.0291)/2.

[46]$0.70(1-0.0398)/(0.30(1-0.0398))$.

[47]0.5(0.29) + 0.5(0.67).

[48]0.52(0.00) + 0.48(0.480).

[49]$0.52(0.00 - 0.23)^2 + 0.48(0.5(0.29 - 0.23)^2 + 0.5(0.67 - 0.23)^2)$

is smaller than in Model 9A. As in $N = 2$, return and risk are both smaller once we take shipping cost into account.

We can then compare this combination of return and risk with those achievable at other sizes of market. I repeated the same process of calculating $U[N]$ and $V[N]$, as described above, for all market sizes from $N = 2$ to $N = 30$ in the spatial case. See Fig. 9.6. There, I show the risk-return curve as a faint line joining achievable combination at each market size in Model 9A (reproduced from Fig. 9.3) and as a solid line joining dots (labeled Model 9B).[50] I have also reproduced the risk-return tradeoff from Fig. 9.3. For the risk-return curves in Models 9A and 9B, I have labeled selected market sizes from 2 to 30. Here, we see the effects of introducing shipping cost.

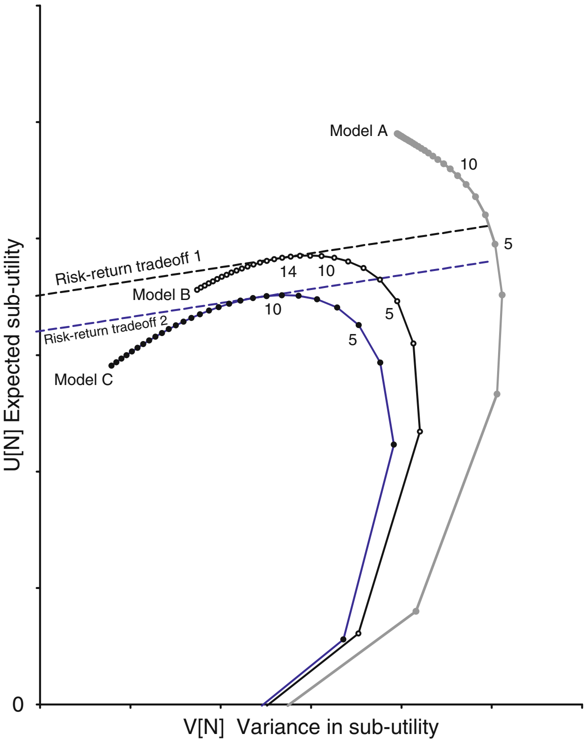

Fig. 9.6 Model 9B: $U[N]$, $V[N]$, and size of market in a spatial market: $\alpha = 0.30$, $\phi = 0.40$, $g = 30$, and $s = 0.40$.
Notes: Market size, N, shown as *dots* for selected N from 2 to 30. *Gray line* is the $U[N] - V[N]$ curve when shipping costs are zero. *Solid curve* is the $U[N] - V[N]$ curve for a marginal individual taking into account higher shipping costs to reach larger market. *Horizontal axis* scaled from 0.04 to 0.10; *vertical axis* scaled from 0.2 to 0.5

[50]In Model B, the shape of the risk-return curve is sensitive to the unit shipping rate, s. As s approaches zero, the risk-return curve approaches that for Model A in Fig. 9.6. On the other hand, as s is made larger, the risk-return curve for Model B is pulled even further back and down at larger N.

1. When $N = 1$, there is no effect since each farmer is in autarky.
2. For $N \geq 2$, shipping cost pulls the risk-return curve downward and to the left: i.e., lower return and lower risk once shipping cost is incorporated.
3. For $N \geq 2$ but small, the introduction of shipping cost reduces both risk and return by a relatively small amount.
4. For $N \geq 2$ and large. Previously in Model 9A, a larger market meant unequivocally lower risk and higher return. In Model 9B however, this is no longer true. When market size is large, the reductions in both risk and return are relatively large. The geography of a larger market area may mean that the cost of shipping eventually reaches a magnitude where it undermines the advantage of further increasing market size.

In the hypothetical example shown in Fig. 9.6, in the presence of shipping costs, the marginal farmer with a β associated with the risk-return tradeoff line displayed would find it best to participate in a cooperative of 14 farmers. In this way, shipping costs help explain why market size is not unlimited. Model 9A suggests farmers have an unlimited appetite for participating in large markets. However, shipping costs in the form of membership fees curbs this appetite. Size of market reflects the offsetting influences of return and risk on the one hand and average shipping costs on the other. If farmers have a higher β—that is, a greater aversion to risk—the risk-return tradeoff line in Fig. 9.6 would be steeper, and farmers would then opt for a coop with a larger N. See (9.4.7) and (9.4.8). In other words, farmers would then prefer a lower level of risk even though that might involve a substantially lower return.

Earlier, I had said that farmers with a similar β would form a club. As is evident from Fig. 9.6, that is not always the case. If s is sufficiently small, every farmer would prefer an infinitely large market, and we might therefore find a great mix of βs among farmers in any club so formed. It is more correct to say that if s is sufficiently large, the risk-return curve under Model 9B will consist of a set of distinct market sizes each of which will make that size best for farmers within an interval of β. In that sense, Model 9B is lumpy. I draw the risk-return curve as a polyline in both Figs. 9.3 and 9.6; however, this is just for ease of exposition. In fact, the risk = return curve is just a set of combinations of $U[N]$ and $V[N]$: one for each integer size of market; the line segments joining them have no particular meaning. When, in a beta analysis, we draw a risk-return tradeoff as an upward sloped line, we are asking simply which combination of $U[N]$ and $V[N]$ on the risk-return curve allows the farmer to reach the highest risk-return tradeoff. In the example shown in Fig. 9.7 for instance, farmers whose β is above 0.0873 would choose at least $N = 12$ (since that is the slope of risk and return joining $N = 12$ to $N = 11$); farmers whose β is below 0.3168 would choose no more than $N = 12$ (since that is the slope of risk and return joining $N = 12$ to $N = 13$). In Fig. 9.8, I show a step function from which we can read, for any given β, the appropriate size of market in this example; for instance, at $\beta = 1.4$, the farmer chooses a market of 20 in Model 9B. Others prefer to tell economic stories without such step functions (lumpiness); they would like something akin to Model 9B but wherein size of market, N, was a continuous

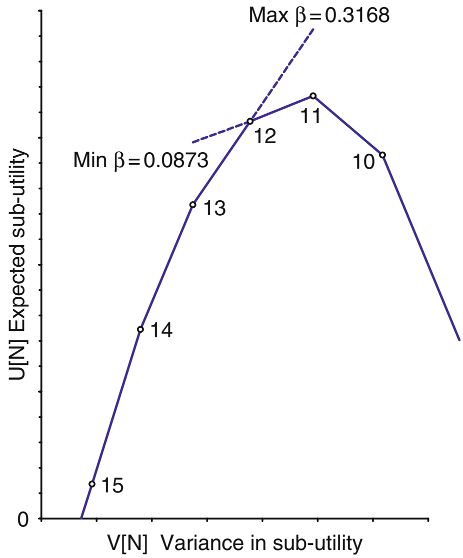

Fig. 9.7 Model 9B: Risk and return by size of market. *Notes*: $\alpha = 0.30$, $\phi = 0.40$, $g = 30$, and $s = 0.40$. *Horizontal axis* scaled from 0.065 to 0.072; *vertical axis* from 0.391 to 0.393

variable that could be analyzed more easily using calculus or other methods that rely on continuity.[51] However, I like Model 9B from a pedagogical perspective because it clarifies just how a farm might decide in practice whether to join a given coop.

What about comparative statics in this model? First note that compared to models elsewhere in this book Model 9B has relatively few parameters: ν, β, ϕ, g, and s.

- g In Model 9B, g appears to be measuring something similar to s. As s and thereby f is increased, the loss associated with the cost of shipping increases. As g is decreased, the density of farms declines and the marginal farmer has to travel further to participate in a market of size N and thereby f is increased. Put differently, the loss associated with shipping (and hence the coop fee) increases when g is decreased.
- s In the non-spatial world of Model 9A, a beta analysis of risk and return would lead farmers to congregate in a single global market since the risk-return curve bends up. If s is close enough to zero, that may also happen in Model 9B. In Model 9B, s must be sufficiently large to cause the risk-return curve to have a positive slope (bend back down) for sufficiently large N before we will observe farmers choosing to form a club—i.e., a local (smaller) market. Put differently, if we decrease s and thereby f, the farm has an incentive to

[51] For example, Models A and B here are built on binomial probabilities that arise because we have characterized market formation as a Bernoulli Process. It is well known that the Normal Distribution (which is continuous) can approximate a Binomial distribution as the number of trials in the Bernoulli process (in this case, the size of market) becomes sufficiently large.

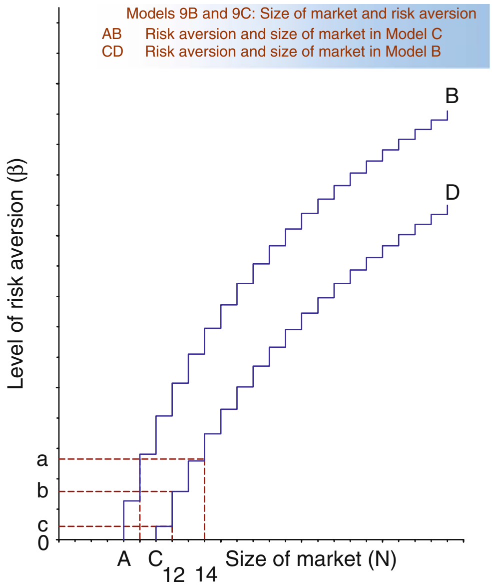

Fig. 9.8 Risk aversion and size of market in Models 9B and 9C.
Notes: $\alpha = 0.30$, $\phi = 0.40$, $g = 30$, and $s = 0.40$. Here, b and c are the lower and upper limits on β in Model 9B when the best size of market is $N = 12$: see Fig. 9.7. For farmers for whom $\beta = a$, best size of market is 10 in Model 9C and 14 in Model 9B. *Horizontal axis* scaled from 5 to 30; *vertical axis* from 0 to 3.5

travel further to benefit from a larger market, and for a sufficiently small s and thereby f, there will be a single global market.

β If β is increased, farmers become more risk averse. They are willing to spend more to join a larger coop because they attach more importance to reducing risk. Provided s is sufficiently large to make the risk-return curve bend back down enough so that a risk-return tradeoff line can be tangential to it, an increase in b causes farmers to prefer a larger market.

ν If ν is increased, each farmer prefers more wheat relative to corn. This causes CEER to fall since farmers now see wheat as more valuable. However, it has no effect on the efficient size of market, N.

ϕ If ϕ is increased, a farmer is more likely to have a corn endowment. Effects on the model depend on the size of ϕ relative to $1 - \nu$. If ϕ is smaller than $1 - \nu$, an increase in ϕ brings the ratio of corn to wheat endowments closer to what consumers would prefer. If ϕ is larger than $1 - \nu$, an increase in ϕ pushes the ratio of corn to wheat endowments higher than what consumers would prefer. The size of market is unaffected.

Model 9B is the first place in this book to make use directly of the idea of a club. Upon reflection, it might seem strange to introduce the idea of a club—whose members cooperate—into a book on the effects of competition (often thought to be the antithesis of cooperation). Why do farmers here seek to cooperate? What is driving them here is the risk associated with the randomness of the harvest—i.e., their endowment. To form a club, as is done in Model 9B, is just one possible response to that risk. What might be some other possibilities?

- *Crop insurance*. If farmers need to consume both corn and wheat, and plant both crops, it would be reasonable for them to purchase insurance against crop failure. Model 9B does not include this possibility, so the alternative is to band together with other farmers hoping that there will be sufficient amounts of both wheat and corn at the market to enable a good trade. Presumably, crop insurance is structured to give farmers (every farmer in this case) an amount of the commodity they fail to harvest. Presumably the insurer collects a premium from each farmer, awards every farmer an insurance benefit, and still manages to keep something for himself or herself (e.g., profit and wages for the insurer and his or her workers). Under Model 9B, each farm pays its share of shipping costs (which is like an insurance premium). Each farm also receives back some amount of the commodity that they did not harvest (except when $k = 0$ or $k = 1$). In that sense, the farmers in Model 9B have greater risks than they would with crop insurance (since they could still arrive at a market where k was near 0 or near 1 but less risk than they would have in the absence of both a commodity market and crop insurance).
- *Arbitrage*. Model 9B assumes there is no way to purchase commodities other than by traveling to a market. As envisaged here, there would potentially be a large number of local markets each with its own exchange rate between wheat and corn. Where exchange rates differ between two local markets by more than the unit shipping cost between markets, presumably arbitrageurs would have an incentive to enter. If the farmer knows that there are competitive traders lurking in every market looking for such opportunities, they will have an incentive to choose a smaller market and save on the cost of shipping to the extent that they know traders will act in a way that keeps the local exchange rate from becoming too unfavorable.

At the outset of this book, I suggested that the organization of the firm was itself endogenous to locational competition. We have seen illustrations of that idea earlier in the book, most recently in the outsourcing decision in Chapter 7. If we think of farmers as firms, and the coop as an extension of the activities of a firm, Model 9B offers a new insight in this matter. With the creation of the coop, each farmer is agreeing to be bound by conditions and restrictions of that group. In effect, the farm's pursuit of its own well-being is now spread across two establishments: one at the farm site and one at the level of the cooperative (market). In effect, the coop helps each farmer achieve a better market outcome than they might otherwise have.

In the Marshallian sense, can Model 9B be thought of as a localization economy? I think the answer to that is yes. In Model 9B, the coop spreads (shipping) cost among participants and clusters farms. The process is endogenous; farmers participate in a cluster (or not) not by fiat but on the basis of their own assessment of costs and benefits. Here although farmers close to the market point pay a greater co-op fee than they recoup in shipping cost reimbursed, there is no incentive to leave the coop. Even though they are cross-subsidizing the more remote farmers, they know that the coop has pushed those farmers into concentric rings that are the most efficient and that they could not get the efficiencies of a market of this size without the coop.

Let me expand on the idea of a club here. As presented, the coop internalizes the externality of market formation by sharing shipping cost and ensuring efficient location. In a more general sense, local government generally can be thought to (1) address local externalities through infrastructure investment, service provision, and activity regulation and to (2) redistribute costs among municipal revenue sources such as property tax, local sales tax, and other sources in addition to user fees. In this sense, a club can be thought of as a metaphor for local government. Put differently, coops serve to separate farmers by risk category; local governments can perform a similar function.

9.6 Model 9C: Competition for Land in a Spatial Market

Now, consider a model in which there is no cooperation among farmers. Instead, assume farms choose a market and bid for a location in proximity to that market in the knowledge that each must pay their own shipping cost to get their endowment to the market. See (9.5.1) and (9.5.2) in Table 9.5. As in Model 9B, I assume that unit shipping cost is contingent on distance to the market chosen and is independent of the exchange rate that gets established for wheat and corn. In a competitive process, I imagine that market rent for land rises above the opportunity cost of land (paid by farmers implicitly in Model 9A and Model 9B) insofar as scarcity rents arise around market points. In this model, landlords are again absentee in the sense that rent payments to them disappear and do not affect the exchange rate in any local market. I also assume that all rent payments are contingent; the farmer's rent is a fixed proportion of his or her endowment regardless of the exchange rate that gets

Table 9.5 Model 9C: competitive farmers and the market in a spatial economy where r is endogenous

Outer boundary of farm at ring i from market $x_i = \surd(i/(\pi g))$	(9.5.1)
Mid distance for farm at ring i from market $m_i = \surd((i - 0.5)/(\pi g))$	(9.5.2)
Endowment for farm at ring i from market net of shipping cost $(1 - r/g - sm_i, 0)$ or $(0, 1 - r/g - sm_i)$	(9.5.3)
Rent at farm N at market boundary $r[N] = 0$	(9.5.4)
Rent at farm closer to market $r[i] = s\surd(g/\pi)\{\surd(N - 0.5) - \surd(i - 0.5)\}$	(9.5.5)

Notes: See also (9.1.1) through (9.1.6) I and Table 9.1. *Rationale for localization* (see Appendix A): Z4—Risk spreading and insurance; Z6—Differences among consumers; Z7—Variation in goods; Z8—Limitations of shipping cost. *Givens* (parameter or exogenous): g—*D*ensity of farms; i—*P*osition of farm: rings away from the market; N—Number of farmers in market. *Outcomes* (endogenous): m_i—Mid-radius of farm i; r—Market rent; x_i—Outer boundary of farm i.

established in any local market. In Model 9C, I retain the assumption that farms each form a concentric ring around the market point. After all, shipping cost now gives the farm an incentive to want to be near the market point. Between any pair of farm sites (rings), competitive bidding for land means the difference in rent in equilibrium must be just enough to make the farmer indifferent between the two sites. Rent for the farmer at the boundary of the market area is zero; see (9.5.4). Further, since I assume that the amount of land used by a farmer is fixed, the equilibrium difference in rent (per harvest) between the two sites must exactly offset any savings in the cost of shipping. That rent plus shipping cost must therefore total to a constant is the so-called Wingo condition.[52] Rent for the farmer in ring i is given by (9.5.5). Once scarcity rents reach this level, there is no incentive for a farmer to prefer any one ring to another within a given market. However, there will still be differences among markets of different sizes. The implication of (9.5.5) is that the larger the N, the higher the rent in ring 1.

Given a market of size N, there are important differences between Models 9C and 9B. In Model 9C, the farmer at ring N loses from his or her endowment a shipping cost of $s\surd((N-0.5)/(\pi g))$, zero rent, and (needless to say here in Model 9C) there is no coop fee. In Model 9B, the same farmer incurs zero shipping cost, zero scarcity rent, and a coop fee of $(1/N)\Sigma_i s\surd((i-0.5)/(\pi g))$. At ring N, the loss to endowment is smaller in Model 9B than in Model 9C. At any location closer to the market, the loss to endowment for the farmer in Model 9B stays the same and so too does the loss to endowment for the farmer in Model 9C since rent there increases to offset any savings in shipping cost. Therefore, the farmer in Model 9B is better off than the farmer in Model 9C by the same amount regardless of location in a market of a given size. Why is this? It is because absentee landlords are collecting scarcity rents in Model 9C that do not appear in Model 9B.

To begin an interpretation of Model 9C, consider panel (c) of Table 9.2 wherein $N = 2$. Suppose $N_1 = 1$. The wheat farmer has an initial endowment (income) of (1, 0). Continuing the assumption that $\nu = 0.3$, this person prefers to consume 30% of their endowment (after rent and shipping cost) of wheat and trade the remaining 70% away for corn. The corn farmer has an initial endowment of (0, 1) of which he or she prefers to consume 70% (again after rent and shipping cost) and trade away the remaining 30% for wheat. Since each farmer, regardless of location, pays the same total of rent and shipping cost, that exchange ratio between wheat and corn in this market is still 2.33 units of wheat per unit of corn just as in Models 9A and 9B. There are two possible utilities when $k = 0.5$: a (1, 0) endowment that yields a consumption bundle (0.28, 0.28) and a utility (U_1) of 0.28 (compared to 0.29 and 0.30 in Models 9B and 9A respectively) with a probability (k) of 0.5 or a consumption bundle (0.66, 0.66) and a utility (U_2) of 0.66 (compared with 0.67

[52] Wingo (1961) originated the idea that land rent offsets transportation cost savings. Later, Alonso (1964) argued that the relationship between land rent and transportation cost savings was also affected by the elasticity of substitution between land and other commodities. Since I have here assumed that the amount of land used by each farmer is fixed, I do not have to take elasticity of substitution into account.

or 0.70 in Models 9B or 9A respectively) with a probability $(1 - k)$ of 0.5. Then, $W[k] = 0.475$[53] (compared to 0.48 or 0.50 in Models 9B or 9A respectively). The introduction of shipping cost here leaves the exchange rate unchanged but reduces the sub-utility in every outcome, not surprising given that rent and shipping cost reduces the amount of wheat and corn available for consumption. As a result, $U[N]$, from columns [4] and [11] in panel (b) of Table 9.2, is 0.228,[54] down from 0.230 or 0.240 in Models 9B or 9A, respectively. Further, the variance in this utility, $V[N]$, is also now calculable; $V[N] = 0.074$[55] down from 0.075 or 0.082 in Models 9B or 9A, respectively. To conclude, in the case of $N = 2$, the introduction of rent and shipping costs reduces both return and risk compared to Models 9B and 9A.

Now let us do similar calculations for a spatial market of $N = 8$ participants: see panel (c) of Table 9.3. CEER remains the same as in Models 9B and 9A. However, compared to Models 9A and 9B, we find—as when $N = 2$—that U_1, U_2, and $W[k]$ drop for any $0 < k < 1$. As a result, $U[N] = 0.375$ is smaller than it was in either Models 9B or 9A. $V[N]$ too is smaller than in either Models 9B or 9A. As in $N = 2$, return and risk are both smaller once we take rent and shipping cost into account. A further implication is that, in a risk-return analysis, the farmer would prefer $N = 8$ over $N = 2$ in Model 9C.

We can then compare this combination of return and risk with those achievable at other sizes of market. I repeated the same process of calculating $U[N]$ and $V[N]$, as described above, for all market sizes from $N = 2$ to $N = 30$ in Model 9C. See Fig. 9.6. There, I show the risk-return curves for Model 9C in addition to Models 9B and 9A. Here, we see that the effects of introducing rent and shipping cost is to cause the risk-return curve to bend back down (have a positive slope when N is sufficiently large) even more than in Model 9B. In the example shown in Fig. 9.6, in the presence of rent and shipping costs, the marginal farmer with a β associated with the risk-return tradeoff line displayed would find it best to participate in a coop a market of just 10 farmers. In this way, rent and shipping costs help explain why market size is not unlimited. Model 9A suggests farmers have an unlimited appetite for participating in large markets. Model 9B shows that shipping costs curb this appetite. In the presence of land rent, the efficient size of market shrinks even further in Model 9C where competition rather than cooperation is seen to underlie a firm's behavior.

In the example of Model 9C illustrated in Fig. 9.6, firms with the β illustrated by the slope of the risk-return tradeoff choose $N = 10$, compared to $N = 14$ in Model 9B. In Fig. 9.8, I show that the best size of market is consistently lower in Model 9C than in Model 9B at every level of β. The reason is simple; the marginal farmer in Model 9C finds it more costly to participate in a market of a given size (because they pay the full shipping cost from the edge of the market area) compared to the marginal farmer in Model 9B (who pays only the average shipping cost) and

[53] 0.5(0.27) + 0.5(0.66).

[54] 0.52(0.00) + 0.48(0.475).

[55] $0.52(0.00 - 0.228)^2 + 0.48(0.5(0.28 - 0.228)^2 + 0.5(0.66 - 0.228)^2)$.

therefore chooses a smaller size of market at any given level of risk aversion. In effect, the mechanism of land rent serves to separate farmers by risk category just as do coops.

It is typically argued that competitive markets are efficient. Is that always the case here? The answer—perhaps surprisingly—is no. Specifically, a farm with a β corresponding to the risk-return tradeoff shown in Fig. 9.6 would be better off in Model 9B than in Model 9C. Put differently, the outcomes in the cooperative scheme in Model 9B appear to be more efficient than in the competitive scheme in Model 9C. Why is this? Two related answers come to mind. First, Model 9B may only appear to be more efficient because it ignores the costs of organizing cooperatives. If such costs were substantial, they could drag the risk-return curve for Model 9B below that for Model 9C, making the competitive solution the more efficient. Second, under the cooperative scheme, farms organize themselves into a larger unit to reap the advantages brought about by the reduction of risk. In this sense, the Coasian view would be that the firm is merely internalizing something (here, a reduced risk) that might otherwise be unavailable or costly to provide in a competitive market. Put differently, belonging to a collective is integral to firm organization: allowing the firm to be efficient. A similar idea arose in the modeling of repair services within or outside the firm back in Chapter 7. There too the firm sought to deal with risk (in that case, machine failure) efficiently through firm organization: be it in-house or through outsourcing.

Models 9B and 9C say something new about why individuals participate in a local market. In earlier chapters, I presented the notion of range of good wherein size of market was limited only by the notion that a commodity was expendable. In Chapter 8, we saw a market boundary defined by the criterion that the presence of a competitor nearby limited the ability of a firm to attract customers. In this chapter, the farmer can be thought to make a choice between a smaller market and a larger market in terms of the of risk and return associated with each.

This is the first model in the book wherein land rents vary by location within the regional economy. We can think here of a farm at the edge of its market area (i.e., the Nth distant farmer in a market of N farms) as a marginal farm. Compared to the marginal farm, the endowments of farms that are closer to the market (i.e., farms 1 through $N - 1$) now can be seen to include an excess utility that arises because of a savings in shipping cost. In Model 9C, the excess utility gets bid away as land rents so that the first $N - 1$ farmers end up only as well off as the marginal farmer in their market. Put differently, these land rents do not arise because of something that landlords have done to make their properties individually more attractive to tenants. Instead, they arise because of the clustering of N farms. To farms, they are a loss in consumption (a leakage to the farm economy) that happens because farms compete in the land market. The model is silent on how farms produce their endowments. However, we could readily imagine the farm as an enterprise that uses capital, labor, and land to produce its wheat or corn. If so, Model 9C hints at a fundamental underlying relationship among factor payments to these inputs. In Model 9C, the gain in utility from participating in a larger market is divvied up between (1) implicit returns to labor and capital in the farm enterprise and (2) the return to land. I present models in later chapters of this book that further explore this idea.

What about the comparative statics of Model 9C? Once we take into account that the coop fee in Model 9B is now replaced by shipping cost and land rent, the comparative statics are much the same as in Model 9B. Why is that? In part, it is because Models 9B and 9C have the same parameters: ν, β, ϕ, g, and s. In Model 9B, the farmer pays a fixed cost (the coop fee) regardless of location that increases with the size of market. In Model 9C, the farmer also pays a fixed total cost (for shipping plus land rent) that increases with the size of the market.

- g In Model 9C, as g is decreased, the density of farms declines and the marginal farmer has to travel further to participate in a market of size N, and thereby shipping cost is increased. Put differently, the loss associated with shipping and/or rent increases when g is decreased.
- s If s is near zero, farmers in Model 9C form a single global market because the risk-return curve bends up. If s is large enough to cause the risk-return curve to have a positive slope (bend back down) for sufficiently large N, we observe farmers choosing to form a local (smaller) club. Put differently, if we decrease s, the farm has an incentive to travel further to benefit from a larger market, and for a sufficiently small s, and there will be a single global market.
- β If β is increased, farmers become more risk averse. They are willing to spend more to join a larger coop because they attach more importance to reducing risk. Provided s is sufficiently large to make the risk-return curve bend back down enough so that a risk-return tradeoff line can be tangential to it, an increase in β causes farmers to prefer a larger market.
- ν If ν is increased, each farmer prefers more wheat relative to corn. This causes CEER to fall since farmers now see wheat as more valuable. However, it has no effect on the efficient size of market, N.
- ϕ If ϕ is increased, a farmer is more likely to have a corn endowment. Effects on the model depend on the size of ϕ relative to $1 - \nu$. If ϕ is smaller than $1 - \nu$, an increase in ϕ brings the ratio of corn to wheat endowments closer to what consumers would prefer. If ϕ is larger than $1 - \nu$, an increase in ϕ pushes the ratio of corn to wheat endowments still higher than what consumers would prefer. The size of market is unaffected.

Before leaving this section, let me add one thought about landlords. As envisaged here, there will be a mix of markets across the landscape; some small and some large. Corresponding to these will be a mix of landlords; some will receive a high scarcity rent for their site because it happens to be near the center of a large market; others will receive only a rent equal to the opportunity cost of the land: i.e., a zero scarcity rent. If landlords themselves were competitive, why would a landlord with a zero scarcity rent not seek to entice farmers to relocate more advantageously? In the absence of collusion and given enough landlords (who each own only a small amount of land), there is no mechanism by which this could be achieved. At the same time, the potential for a higher land rent might in practice create an incentive for landlords to collude locally to ensure that they attract and hold the scarcity rents associated with a larger market.

9.7 Final Comments

In this chapter, the principal model has been 9C. I included Models 9A and 9B to help readers better understand aspects of Model 9C. In Table 9.6, I summarize the assumptions that underlie Model 9A through 9C. Many assumptions are common to all these models: see the list in panel (a) of Table 9.6. The models differ in that (i) Model 9A assumes shipping costs are zero, (ii) Model 9B assumes that farms address differences in shipping costs with location by forming cooperatives, and (iii) Model 9C assumes that farms address differences in shipping costs by bidding up the price (rent) for land at advantageous locations.

Table 9.6 Assumptions in Models 9A through 9C

Assumptions		9A [1]	9B [2]	9C [3]
(*a*) *Assumptions in common*				
A1	Closed regional market economy	x	x	x
A2	Barter market	x	x	x
B1	Exchange of wheat for corn	x	x	x
C10	Two kinds of customers	x	x	x
C7	Maximize same utility function	x	x	x
F6	Firm is myopic	x	x	x
G2	Uncertainty explicit in model	x	x	x
G3	Risk incorporated into utility function	x	x	x
(*b*) *Assumptions specific to particular models*				
E1	Zero shipping cost everywhere	x		
A4	Rectangular plane		x	x
J3	Each farm occupies the same amount of land		x	x
F8	Co-operative shares shipping costs among members		x	
J1	Zero opportunity cost (rent) for land			x
J6	Competitive market for land			x

I end this chapter with six sets of thoughts.

First, in Chapter 1, I introduced the idea of the importance of transaction cost in location theory. I defined there the concept of an effective price which includes the price paid plus the unit transaction costs incurred by the purchaser related to search and information gathering, negotiation, and acquisition, inclusive of normal profit. In Model 9B here, the coop fee can be thought of as a transaction cost. In Model 9C, the combination of shipping cost and rent can similarly be thought of as the transaction cost. In both models, the farmer is free to choose a level of transaction cost (i.e., a size of market or location). At the same time, there is nothing to guarantee that the farmer will always get a better price in a larger market. In these respects, choosing a larger market is like searching. The farmer in a larger market is getting to see a wider cross section of market participants. This is like (but not the same as) a consumer gathering information from different suppliers before deciding whether and from whom to purchase. The reader might therefore be tempted to apply this model to help in understanding search behavior. However, there are important differences between market formation as modeled here and conventional thought about search behavior. Principal among these in my mind is the idea that search is often seen as a sequential process. In my view, the consumer gathers information about another supplier or commodity, then makes a decision about whether to continue searching, to purchase, or give up on the idea of purchasing such a commodity. In contrast, the farmer in Models 9B and 9C is simply making a gamble among sizes of market; there is no sequential process at work here.

Second, the farmer in this chapter is always assumed to use the same amount of land in production. However, in Model 9C, land becomes relatively more costly to rent, the closer the farm is to the market. Presumably, when an input like land becomes more costly, we might expect the farmer to change the way in which he or she produces agricultural commodities. One simple way to do this is to switch between production that is less labor intensive (when land rents are low) and production that is more labor intensive (when land rents are high). To do this, our model of the farm would have to include a production function that enables a substitution between land and labor. The models in this chapter assume a fixed amount of land per farm and are silent on labor input. From my perspective, the easiest way to think about the farmer in this chapter is that he or she constitutes one unit of labor and that the farm has a Leontief production technology that requires exactly 1 unit of labor and $1/g$ units of land to produce stochastically an endowment of either (1, 0) or (0, 1). What happens if we make amount of land used by a farm endogenous to the model? I return to this question over the next three chapters.

Third, we began Model 9A with the apparently innocent assumption that each farm had an independently drawn random endowment of 1 unit of either wheat or corn. Implicit in this assumption is the idea that the scale of a farm is somehow fixed. Imagine how much different the model might look if we started with the assumption that the farm had an independently drawn random endowment of 1 unit of either wheat or corn for each unit of land farmed and was free to hire labor and rent land as needed. In that case, a farm could largely (if not entirely) eliminate the need to travel to a market by being of sufficient size to render insignificant the risk of being without suitable quantities of the two goods. In effect, the farm uses its own size to self-insure rather than rely on what is essentially an insurance function performed by the market. The gain to the farmer here is the higher level of utility now possible in the absence of scarcity rents.

Fourth, I am able to solve the models in this chapter because I took a Walrasian perspective on exchange rates: i.e., I found the exchange rate that leaves neither kind of farmer feeling that there is still some amount of one commodity that they would prefer to trade for the other. However, as noted at the outset of the chapter, the Walrasian solution is just one point on a Marshallian Contract Curve. At one level, one might argue that this indeterminacy of outcome makes the Marshallian approach less useful than Walrasian approach. One might also argue that, as the number of market participants becomes large, the Marshallian solution converges on the Walrasian solution. Nonetheless, it seems to me unreasonable to believe that farmers will entirely ignore the prospect that in a small market their bargaining power (however derived) may leave them at an effective exchange rate different from that envisaged by Walras. The models in the chapter do not consider this.

Fifth, what about Walrasian equilibrium across markets? In Model 9A, there is only one market: a global market for the exchange of wheat for corn. Walrasian equilibrium is irrelevant here. In Model 9B, unit shipping costs drive farms to organize themselves into local markets for the exchange of wheat for corn. Because the exchange rate realized in each local market is subject to random variation, we can say only that this particular process of choosing local markets leaves individual farms best off viewed over a longer term of repeated trials. The same is true of Model 9C with a twist; in 9C, there is a market for land as well as local markets for the exchange of wheat for soap.

Sixth, Chapter 9 gives us some hints about the regional economy. However, much remains to be done to flesh this out. We need to know more about the process by which endowments of wheat and corn are generated. It would be helpful for example to know about the use of labor, capital, and land inputs in the production process. That would make it possible for us to better understand the regional economy and the impacts of changes in givens on regional well-being.