Price Behavior on a Declining Terminal Market

William G. Tomek

The relationship of prices of choice steers on the Denver terminal market to comparable prices on the Omaha market apparently was influenced by the sharp decline in saleable receipts in Denver. But price effects of the volume of trading were measurable only when the volume in Denver became extremely small in 1967 and 1968. These results are consistent with defining a thin market place by using sampling concepts. Some subtleties of using Chebyschev's inequality to define a thin market are discussed.

Key words: Chebyschev's inequality, markets, price discovery, sampling, steer prices, thin markets.

Thin markets may create problems in pricing farm products. One concern is that a small volume of trading at a central market place can result in price behavior not warranted by economic conditions. Moreover, deliberate manipulation of prices is more feasible with a small volume. If the central market quotations are used as base prices in other transactions, the problems of unwarranted or manipulated prices acquire increased economic importance.

Research and writing on thin markets can be categorized under five questions. First, what is a thin market? Second, why do markets become thin? Third, if a market becomes thin, what are the consequences, if any, for price behavior? Fourth, if price behavior is influenced, what is the mechanism or linkage between market volume and price behavior? Finally, if thin markets have adverse consequences, what solutions exist for the problem? Although a considerable general literature exists on thin markets (e.g., Hayenga), there is little specific information or empirical evidence on these questions. In this context, this paper has two objectives: to develop a more precise definition of thinness and to investigate the effects on price behavior of a declining volume of trading on a central market. In addition, the linkage between volume and price behavior is discussed briefly.

The empirical application is for choice steer prices in Denver, a terminal market that declined and disappeared. Steer prices in other locations are used as a standard of comparison. This paper, however, is essentially a search for hypotheses and an exploration of methods, and the empirical results should not be interpreted as definitive conclusions about price behavior on thin markets.

Defining and Analyzing Thin Markets

A major concern about thin markets is that the number of transactions (per unit of time) is so small that "unwarranted" price behavior occurs. The warranted price is usually defined in terms of the perfect competition norm. Prices may deviate from the norm because of deliberate manipulation or poor information. The issue of manipulation, however, is difficult to analyze with the data typically available. Observations on the actions and motives of individual traders are probably needed.

The issue of poor information perhaps can be analyzed with existing, secondary data. A declining volume of transactions implies less information and perhaps lower quality information. In a perfect market, a single equilibrium price exists for a given quality product at one location at a point in time (say, a day). A single transaction price, however, contains

William G. Tomek is a professor of agricultural economics, Cornell University.

The paper was written while the author was a visiting economist, Economics, Statistics, and Cooperatives Service, U.S. Department of Agriculture, and their support of the research is gratefully acknowledged.

M. L. Hayenga, J. Hassler, R. G. Heifner, K. H. Kahl, J. S. Mann, K. E. Nelson, and T. D. Mount provided helpful comments on an initial draft. The author, of course, accepts full responsibility for the final content.

some error relative to the unknown equilibrium; the error may be large or small; the magnitude is unknown. An increase in the number of transactions within the time period increases the information available. Conversely, as volume declines, the quantity of information declines.

Information may be defined statistically as the reciprocal of the variance. If, as is assumed below, the average of transactions prices is an estimate of the true equilibrium price, then the variance of the mean of transactions prices decreases as the number of transactions increases (but see note 1). In this sense, the intuitive notion that fewer, publicly reported prices reduce information is consistent with statistical theory.

In addition, the quality of information may decrease. For example, a meat packer maintains a knowledgeable buyer at a market place with a large volume. The buyer has available the resources of the firm as well as his experience. With a declining volume, the cost per transaction of maintaining an experienced buyer at a particular location increases, and this buyer may be withdrawn. (Of course, other reasons may exist for the processor leaving the market.) Thus, the quality of information probably is reduced, and this poorer information may be translated into poorer price behavior. This argument does not necessarily suggest a bias in prices, but rather, larger errors in price formation.

Much of this paper deals with changes in prices with the passage of time and differences in prices among locations. The price discovery process is one of finding the warranted adjustments from existing levels as expectations about factors determining price change. Price changes are thought to follow a random walk or martingale process in a perfect market (Samuelson); but, for the purposes of this paper, the question is how may the number of transactions influence the discovery of warranted price changes? Intuitively, this number may not be a constant. An unexpectedly large change in supply, for example, may require more transactions to discover the correct change in price than does a small change in supply.

Defining a Thin Market

Chebyschev's inequality perhaps can help provide a specific definition of a thin market. One statement of the inequality (e.g., Cramer or Mood) is

$$P(|X_n - \mu| \geq c) \leq \frac{\sigma_n^2}{c^2},$$

where P defines the probability of the expression, and c is an arbitrary constant. X_n is a random variable with mean μ and variance σ_n^2. The subscript n is added to emphasize that X_n and σ_n^2 depend on n. For example, X might be an individual transaction price. Hence, X_n would be the mean of transactions prices for particular time period and σ_n^2 the variance of the distribution of the mean. One of the advantages of using Chebyschev's inequality is that it is applicable to distributions of random variables with finite variances. The results do not require that the distribution of prices or price differences be normal.[1]

In the inequality, μ can be treated as the true, but unknown, equilibrium price for given economic conditions (though this is not the definition used in most of the paper). Two views exist in the literature about the relationship of actual transactions prices X to the equilibrium μ. One is the $E(X) = \mu$; the mean of observed prices is treated as an unbiased estimate of the equilibrium. Some mixed evidence, based on experiments, exists about this assumption (Hess, Smith). In any case, the assumption implicit in most econometric models is that the average reported prices are equilibrium prices.

The tâtonnement process is a second concept for relating the X's to μ; prices progressively converge toward equilibrium (for a recent review, see Paul). Then, X_n is the last price in a series of n transactions and is the estimate of equilibrium. Again, one can ask, how many observations are needed to obtain a "precise" estimate of the population param-

[1] This paper does not discuss distributions of prices, but the topic is important for estimating the number of reported transactions needed for accurate pricing. Distributions of price differences on futures markets may not have finite variances (Mann and Heifner), but Samuelson hypothesizes that the variances of price differences on futures markets increase as the maturity of the futures approaches. In any case, this paper deals with changes in cash prices. Cash prices typically have been assumed to be distributed normally with constant variances. But the assumption of normality is based on an appeal to the Central Limit Theorem rather than on empirical analyses, and the analysis in the text suggests the variance is not constant. If the assumption of a normal distribution is accepted, then the probability of a particular level of precision could be greatly reduced relative to Chebyschev's inequality. For example, if $c = 2\sigma_n$, then by the inequality $P \leq .25$. For the normal distribution, $P \leq .05$. The use of Chebyschev's inequality can be justified as a conservative approach. On the other hand, the normal distribution is often assumed, and with estimated variances, the t distribution can be justified.

eter? A tâtonnement process, however, typically is not observable in real world data, and from a practical viewpoint, the average of transactions prices must be used.

Thus, defining a thin market is equivalent to finding the number of transactions, n, required to obtain a "large" probability that X_n is "close to" μ. To find n, Chebyschev's inequality may be rewritten as

$$P(-c \leq X_n - \mu \leq c) \geq 1 - \frac{\sigma^2}{nc^2}.$$

For the mean X_n to be within the range $-c$ to $+c$ to μ with probability P,

$$P = 1 - \frac{\sigma^2}{nc^2} \text{ or } n = \frac{\sigma^2}{(1 - P)c^2}.$$

Alternatively, if an existing market or pricing arrangement has n transactions (per unit of time) with a particular variance, then the implied precision of pricing in terms of P and c can be estimated.

This framework suggests, first, that the definition of a thin market is somewhat arbitrary. It depends on the definitions of "large probability" and "close to," i.e., on c and P. A second implication is that the concept of a thin market is a relative one, given c and P fixed while σ varies. The number of transactions, n, will vary as σ varies if a fixed standard of precision is used.

From a sampling viewpoint, increasing n increases costs. From a price discovery viewpoint, the cost relation is less clear. Given N total transactions per day, n of them at a terminal market, thereby contributing to public information, little empirical research exists to explain why n is declining, although some obvious hypotheses exist. (It costs market participants less, say, to formula price based on a central market quotation than actually to participate on the central market.) Hence, a question arises about the trade-off between precise prices and the number of transactions, n. This is an area for fruitful research, but cost topics are not explored in this paper.

The discussion to this point has emphasized intraday or daily prices assuming fixed economic conditions. Obviously, with the passage of time, economic conditions and equilibrium prices change. Prices also vary, of course, with quality and space. In this context, the parameter μ can be interpreted as the true price difference (through time, between regions, or between qualities). The remainder of this paper uses, almost exclusively, price changes or differences. The unit of time also is important in defining a thin market. This paper considers day-to-day and week-to-week changes—the time periods usually considered in the thin market literature.

Measuring Performance

Measuring the performance of a thin market is a problem, in part, because the perfectly competitive norm is not observable. There are likely to be problems in developing a usable norm. Price differentials that should prevail under competitive conditions can be computed, and observed differentials can be compared with the computed values. For example, the differences in economic value of different yield grade carcasses can be computed based on the differences in saleable retail meat, and then actual price differences can be compared with the true economic differences (Trierweiler and Hassler).

Equilibrium price levels, of course, are not observable. In principle, an econometric model could be constructed and estimated. If the model were correct, it would provide unbiased estimates of equilibrium prices under stated conditions. However, most econometric models pertain to quarterly or annual observations and not to the daily or weekly prices of interest in thin markets. Moreover, it is always difficult to build correct models and especially difficult to model short-term prices. But, for short-term prices, the random walk model can be used as a norm (Mann and Heifner).

In this paper, prices on the Omaha market are used as a norm in the study of Denver prices. Omaha is, in effect, treated as a proxy for the national market. During the period under analysis, Omaha had a large volume of cattle sales, including choice steers, but of course no market is perfect. Imperfections are a matter of degree. Thus, while differences in performance may exist between markets, they may not be sufficiently large to be measurable by available data and statistical methods. On the other hand, if differences in price behavior are found, they could be due to factors other than differences in volume of trading. While such problems can not be solved, the Omaha market also was compared with the Sioux City market. The volume of choice steer sales remained reasonably large in Sioux City during the sample period while the Denver market expired. Thus, Omaha prices can be compared with another large market and with a declining market place.

Data and Procedures

The Denver central market is potentially an interesting case study. In the mid-1950s, about 850 thousand cattle were marketed per year in Denver; well over 200 thousand were slaughter steers. By 1968, only 10 thousand steers were marketed, and in late 1968, volume was so light that price reports were sporadic. In November 1967, the market shifted to auction pricing of cattle, and the Agricultural Marketing Service discontinued price reporting for steers on the Denver market on 1 January 1969. It closed in 1972. While cattle marketings declined in all central markets especially after 1965, the volume in the late 1960s and early 1970s in Omaha and Sioux City was still greater than in Denver in the 1950s. Clearly the Denver market place became thin by any standard, while the volume in Omaha and Sioux City in the 1950s and 1960s seems adequate for price discovery purposes.

Prices were collected for choice, 1,100–1,300 pound steers at Omaha and Denver, for the 1955–68 period. The same prices were also obtained for Sioux City for selected years within this period. The range of prices on Monday of each week was recorded. Monday has had a large volume of slaughter steers relative to other days of the week. If Monday was a legal holiday, then Tuesday prices were used. Thus, each year has fifty-two or fifty-three observations, the number of Mondays per year.

The availability of prices only as a daily range raises questions about the comparability of the prices between markets. During the sample period, the instructions to market reporters were to give the range between good and prime steers based on market conditions for the day (not necessarily the range of actual transactions). Thus, the intent was for the midpoint of the price range to represent the midpoint of the range of quality of choice steers. The midpoint of the range is used in much of the analysis.

However, for thirteen of the fourteen years in the sample period, the range of daily prices in Omaha was on average wider than in Denver. The Omaha range was especially large during the early years, and it tended to narrow with the passage of time. This is a bit puzzling because market reporters in both locations had similar instructions. Literally interpreted, the data imply that the gap between good and prime steer prices was consistently larger in Omaha than in Denver. Another possible explanation is the difference in reporters' interpretations of the range of choice prices. Given that Denver had relatively few prime cattle, there also may have been few high choice steers causing the Denver reporter to focus upon a slightly lower grade than in Omaha. Some analysis was done using the low of the range on the assumption that these prices may be more comparable in the two markets. The low in Denver tended to decline relative to the low in Omaha. There is no clear trend in the differences of the midpoint, though this difference was quite large the last year of trading in Denver.

The data were plotted and evaluated visually; and the means, standard deviations, and coefficients of variation were computed by year and by month within years. The principal descriptive statistic presented is the coefficients of determination (r^2) for both price levels and first differences.

Regression models were also considered. The Denver and Omaha prices probably should be viewed as simultaneously determined; though, because Omaha is a much larger market, and because Denver is in a later time zone, causation each day may run more from Omaha to Denver than vice-versa. Because of transportation costs, prices in two locations can be somewhat independent of each other; Denver prices could move in a band about Omaha prices defined by transfer costs. Given these considerations, a simple model for the two markets is

$$(1) \qquad D_t = \Pi_0 + \Pi_1 M_t + \Pi_2 X_t + u_t$$

$$(2) \qquad M_t = \lambda_0 + \lambda_1 D_t + \lambda_2 Z_t + w_t,$$

where D is Denver price of choice steers on Monday; M, Omaha price of choice steers on Monday; t, Mondays (weeks); X, predetermined local conditions in Denver; and Z, predetermined local conditions in Omaha.

The problems of fitting equation (1) or (2) are the simultaneity of D and M and whether X and Z can be observed. Obvious candidates for X and Z are the receipts of choice steers in the respective markets on each Monday. Unfortunately these data are not available, though total receipts of cattle are. They were not collected, however, when the price data were obtained.[2]

The Denver-Omaha price differences had a systematic pattern (were autocorrelated), and this probably is related to different patterns of marketings of steers in the two markets. Models that allowed for seasonality or trend were explored, and results for a first difference equation are presented.

$$(3)\quad D_t - D_{t-1} = \gamma + \beta(M_t - M_{t-1}) + v_t.$$

This equation was fitted by year and for selected pooled periods. The intercept can be viewed as a proxy for an omitted variable, and gamma is a measure of regional price differences used in the next section. Although the Durbin-Watson statistics were usually reasonable, equation (3) presumably is a misspecified version of equation (1), and likewise, while causation may run more from Omaha to Denver than the reverse, simultaneous equations bias probably is present in the least-squares estimates. Thus, the estimated parameters likely are biased, though alternative model specifications made little difference in the estimated slope coefficients. With these limitations in mind, emphasis is placed on descriptive statistics.

Empirical Results

An expression derived from Chebyschev's inequality is used to estimate whether or not Denver and Omaha might be considered thin markets. In

$$n = \frac{\sigma^2}{(1 - P)c^2},$$

P and c are selected rather arbitrarily. Emphasis is placed on obtaining an appropriate measure of the variance, σ^2, for the questions at hand. In this paper σ^2 estimated from Omaha prices is used for establishing n related to the change in the price level of choice steers. For Denver, the appropriate σ^2 relates to changes in price differences between Omaha and Denver.

Because the price level need not be rediscovered, a variance based on the first difference of prices is a sensible measure of the variability of the random variable in question. That is, μ for Omaha prices is defined as the change in true equilibrium prices, and σ^2 is estimated from the first differences of observed Monday prices by year.[3] This implies that week-to-week changes in the equilibrium are being analyzed. This variance overstates the difficulty of the market's problem because Omaha prices are available on more than one day per week.[4] In addition, as already discussed, the Chebyschev relation gives a fairly large n relative to assuming a precise distribution of price changes. On the other hand, σ^2 is estimated from market data and is not the true variance. Moreover, a variance based on Monday-to-Monday changes in the midpoint may understate the complexity of discovering the full range of prices from low to high choice steers. The values of n estimated should be treated as the approximations that they are.

For the computations, c was set equal to 10 (cents per hundredweight) and P to 0.9. The value of c is in the context of standard deviations of the first difference of Omaha prices that were often about 50¢ per hundredweight, though the range was from 21¢ to 69¢ for the years in the 1955–68 period. The values for c and P are also convenient in that $(1 - P)c^2 = 10$. Thus, the computed n is just the variance of the first differences of Omaha prices divided by ten. These values are shown in column one of table 1. In interpreting results, the reader should remember that they apply to discovering the price change for 1,100 to 1,300 pound choice steers and that the variance of Monday-to-Monday prices is based on a calendar year sample. In addition, the results are for a specific location, but they are "national" in the sense that they apply to changes in price level and not to changes in regional or quality differentials.

Because the first differences are from Monday to Monday, the estimated n may be inter-

[2] Available time and resources did not permit collection of the quantity observations subsequent to the collection of the price data. Moreover, the addition of seasonal dummy variables or the use of first differences had surprisingly little effect on the estimated slope coefficients, and the specification error bias may be small. Thus, the payoff to collecting market receipts data for total cattle marketings (another proxy variable) is unclear.

[3] The variance is computed from the Monday-to-Monday change in the midpoint of the range of prices. An alternative estimate of the variance could be obtained from the intraday range of individual prices. Assuming a symmetric distribution of prices within the range and given a typical range of $2 per cwt., the approximate standard deviation would be 48¢ per cwt. This value resembles the standard deviations computed from the first differences of midpoints.

[4] Steer prices were usually available on two or three days per week in Denver in the early and mid-part of the sample period, and this number declined to one per week and finally to zero per week. In any case, the collection of daily prices would have greatly increased cost of analysis. Hayenga suggested that one might study market thinness by analyzing prices on days with small volumes.

Table 1. Estimated Number of Transaction Prices for Choice Steers Required to Establish Accurate Price Changes

Year	Price Level Change	Denver-Omaha Difference
	(number per week)	
1955	277	61
1956	337	87
1957	176	70
1958	229	90
1959	193	37
1960	196	49
1961	133	44
1962	224	55
1963	262	65
1964	474	58
1965	243	32
1966	255	42
1967	94	34
1968	45	37

Note: See text for definition of accuracy.

preted as the number of prices (lots or transactions) required for establishing precise week-to-week price changes. The number is not directly comparable with saleable receipts because a typical lot would contain about twenty-five steers (though a lot size down to one is possible). To estimate an equivalent volume in terms of annual saleable receipts, the number of transactions is multiplied by twenty-five animals per lot and by fifty-two weeks. Thus, saleable receipts range from 58,500 in 1968, a year with small week-to-week price variability, to over 616,000 in 1964, a year of large price variability. These receipts are to maintain a precision of plus or minus 10¢ per hundredweight around the true change in mean prices with a 0.9 probability.

In the 1955–68 period, the saleable receipts of choice steers in Omaha ranged from 384 to 570 thousand, and these receipts typically were sufficient to meet the accuracy standard. They are clearly sufficient because the receipts in other markets also help determine the overall level of steer prices. For example, 991 thousand choice steers were sold in Omaha and Sioux City combined in 1964, the year with the largest "requirement" for transactions.

As stated earlier, steer prices are determined simultaneously, but for simplicity, the Omaha-Denver data are used to estimate the number of transactions required in Denver to discover precisely the change in the differential between Denver and Omaha. The differential in question is

$$\mu = (\mu_t^D - \mu_t^M) - (\mu_{t-1}^D - \mu_{t-1}^M),$$

where μ^D is equilibrium price in Denver, μ^M is equilibrium price in Omaha, and t, $t - 1$ is current and previous weeks.

To estimate n, the variance of $\hat{\mu}$ is needed. An estimated difference for a particular week is

$$\hat{\mu}_t = (D_t - M_t) - (D_{t-1} - M_{t-1}), \text{ or}$$

$$(D_t - D_{t-1}) = \hat{\mu}_t + (M_t - M_{t-1}).$$

For a year ($t = 1, \ldots, 52$), an estimate of an average μ and its variance can be obtained from the first difference equation (3), where the notation is changed so that $\gamma = \mu$. The parameter β is thought to equal one, though in practice it may not. In this paper, $\hat{\gamma}$ and its variance are obtained as least-squares estimates of (3).

The level of precision is defined as $P = 0.9$ and $c = 2$, and

$$n = \frac{\hat{\sigma}^2}{(0.10)(4)} = \frac{\hat{\sigma}^2}{.4},$$

where $\hat{\sigma}^2$ varies from year-to-year using equation (3). The level of c is set at a smaller level for the precision of the change in the regional differences than for the change in the price level, but the specific value 2 has no special justification.

The estimated values of n are given in column 2 of table 1, and n ranges from 32 in 1965 to 90 in 1958. As before, n can be interpreted as the number of transactions per week to establish precise changes in regional price differences from week to week. The equivalent annual saleable receipts range from 42 to 117 thousand choice steers. Although data are available only for total steer sales in Denver, receipts in the 1955–66 period appear adequate to discover precise price differences. In 1967 and 1968, however, about 48,000 head of steers would have been required each year for precise price discovery. In contrast, actual receipts were 28,000 and 10,000 steers of all qualities.

Chebyschev's inequality is not needed to demonstrate that the Denver market was thin in its last years, especially 1968. The application, however, does illustrate the complexity of the issue. Moreover, the results suggest that the price effects of a thin market, if any, are likely to be measurable only in 1967 and in 1968. In previous years, volume seemingly was adequate for fairly accurate pricing.

Price Behavior

This paper takes the position that the principal effect of the number of transactions is on the precision of pricing, and a number of measures of precision based on the weekly observations were explored. No simple relation was found between statistics like the range, standard deviation, or coefficient of variation of prices and market volume (e.g., table 2). The effect of volume was obvious in one respect: namely, weeks existed in late 1968 without reported prices for choice steers in Denver ($n = 0$).

The use of r^2, however, does suggest some imprecise price behavior associated with market volume. The coefficient of determination between Omaha and Denver prices was always above 0.81 (for weekly prices by year) from 1955 to 1967, but declined to 0.71 in 1968. For the first differences of prices, r^2 was usually above 0.5 in the 1955–66 period; r^2 for the first differences declined to 0.29 in 1967 and 0.04 in 1968 (table 3).

The first difference equation (3), as discussed, is of interest because the intercept coefficient provides a measure of the average change in regional differentials from week to week by year, and the model is also a way to take account of positive autocorrelation. The slope coefficients of equation (3) with Denver prices dependent tend to decline from the mid-1960s to the termination of the market. The first difference results are summarized by pooling the observations into two groups of years 1955–66 and 1967–68 (table 4). The average slope in the earlier period is 0.79 and in the final two years 0.46. Using the F-test for differences in models over time, the two equations are statistically different at the 5% level; however, given the heteroscedasticity apparent in the residuals of the individual year equations as well as other statistical problems, the hypothesis test should be taken with a grain of salt. Nevertheless, the results for equation (3) and for equations with seasonal dummies (not shown) clearly suggest a changing slope coefficient as market volume declined in Denver.

Table 2. Coefficients of Variation for Choice Steer Prices and Saleable Receipts of Slaughter Steers

	Coefficients of Variation[a]			Number of Steers		
Year	Denver	Omaha	Sioux City	Denver	Omaha	Sioux City
	-----	(%)	-----	---	(1,000 head)	---
1955	7.5	10.7	—[c]	221	864	462
1956	13.9	12.1	—	247	823	428
1957	8.3	7.5	—	192	768	461
1958	4.7	4.5	4.7	172	856	572
1959	3.1	5.1	4.9	169	896	617
1960	3.0	4.9	4.7	151	914	623
1961	5.4	5.7	5.8	163	921	677
1962	5.4	5.8	—	125	933	662
1963	5.6	5.8	—	127	923	639
1964	8.6	8.0	—	165	1,010	657
1965	6.4	6.1	—	104	884	550
1966	4.3	5.2	—	70	799	505
1967	4.3	4.4	4.4	28	704	492
1968	3.5[b]	3.1	2.4	10	586	468

[a] Computed from average price based on midpoint of Monday price ranges.
[b] Based on 43 observations.
[c] Sioux City prices not collected in all years.

Table 3. Coefficients of Determination for Steer Prices

	Price Level Omaha with		First Differences Omaha with	
Year	Denver	Sioux City	Denver	Sioux City
1955	.95	—	.53	—
1956	.96	—	.61	—
1957	.96	—	.20	—
1958	.82	.96	.56	.65
1959	.81	.97	.55	.51
1960	.87	.97	.31	.66
1961	.92	.98	.60	.73
1962	.91	—	.50	—
1963	.90	—	.62	—
1964	.96	—	.81	—
1965	.96	—	.70	—
1966	.91	—	.50	—
1967	.88	.98	.29	.57
1968[a]	.71	.96	.04	.53

[a] Denver observations available only for thirty-eight continuous weeks. Sioux City analysis uses same time period.

Table 4. First Difference Regressions for Pooled Years

Years	$\hat{\gamma}$	$\hat{\beta}$	r^2	d
1955–66	0.145	0.794	0.562	2.49
	(1.402)[a]	(0.028)		
1967–68	0.455	0.463	0.204	2.45
	(2.657)	(0.097)		

$F = 3.016$ (for hypothesis that regression parameters are equal in the two periods, numerator degrees of freedom = 2, denominator = 710).

Note: First difference of Denver prices dependent and first differences of Omaha prices independent variable (¢/cwt.)
[a] Standard error of coefficients in parentheses.

The foregoing evidence, of course, does not prove an association between performance and volume. One way to strengthen or weaken the case for the thinness of the Denver market is to compare Omaha prices with a market that still had a large volume in the sample period. Thus, choice steer prices in Sioux City were analyzed for the years 1958–61 and 1967–68, inclusive. These years were selected because they cover the range of results from the regression analysis of the Denver-Omaha relationship. Not surprisingly, the data show a close association between Omaha and Sioux City prices (tables 2 and 3). The correlations are near one, and the coefficients of variation are similar.

Perhaps the simplest way to compare the Denver and Sioux City markets is through the first difference regressions. The first differences of the midpoint prices for Sioux City are regressed against the first differences of Omaha prices (table 5), and these results can be compared with those for the Denver-Omaha equations. In the 1958–61 period, the Sioux City-Omaha and Denver-Omaha equations are qualitatively similar. In 1961, the hypothesis $\beta = 1$ cannot be rejected for both equations. In 1959 and 1960, the slope coefficients differ from one for both equations.[5] In 1958, the Sioux City equation has a slope different than one and the Denver equation does not, but the slope for the Sioux City equation, like Denver, has a magnitude greater than in 1959 or 1960. The r^2 coefficients are similar. In 1967 and 1968, however, the Sioux City-Omaha equations clearly differ from the Denver-Omaha equations in terms of slope coefficients and r^2's. These results strengthen the view that the precision of pricing in Denver was influenced by declining market volume.

The question of possible bias is more difficult to analyze, but the apparent differences in the seasonal pattern of marketings in the two regions (near Denver and Omaha) were used to explore the issue. In figure 1, the vertical axis is the difference between Omaha and Denver steer prices on average, by month, for the years 1961–65. These years were selected because the regression analyses suggested a close one-to-one relation between Omaha and Denver prices in this period after adjusting for seasonality. (The hypothesis that the true slope equalled one cannot be rejected in the regressions including seasonal dummies.) The horizontal axis of figure 1 measures regional differences in supply, which are approximated by using regional slaughter data for all cattle. Specifically, the sum of slaughter in the Mountain and Pacific regions is subtracted from the slaughter in the Northwest portion of the North Central region (includes Iowa and Nebraska). The dots in figure 1 are the average 1961–65 differences by month, and an inverse relation exists.

The exercise was repeated for the individual months October 1967 to September 1968, the last twelve months with complete weekly price data. This period is represented by the crosses in the figure. Obviously the relationship shifted to the right. For a given level of supply—a fixed difference in slaughter—Denver prices are lower relative to Omaha than in the earlier, base period. This is consistent with the possibility of the Denver market providing prices that are biased downward for given economic conditions. This may be related to the small volume in Denver, but an

Table 5. First Difference Equations, Denver and Sioux City, Selected Years

	Sioux City Price Dependent[a]			Denver Price Dependent		
Year	intercept	slope	r^2	intercept	slope	r^2
1958	−0.072 (3.792)[b]	0.756 (0.079)	.65	0.200 (6.008)	1.021 (0.127)	.56
1959	−0.900 (3.649)	0.605 (0.084)	.51	0.078 (3.856)	0.694 (0.088)	.55
1960	0.920 (2.821)	0.634 (0.064)	.66	−0.093 (4.434)	0.477 (0.101)	.31
1961	−0.526 (2.883)	0.938 (0.080)	.73	1.209 (4.218)	1.009 (0.117)	.60
·						
·						
·						
1967	1.180 (3.005)	0.818 (0.102)	.57	−0.292 (3.662)	0.548 (0.120)	.29
1968[c]	0.330 (2.890)	0.842 (0.132)	.53	2.599 (3.831)	0.216 (0.175)	.04

[a] Midpoint of choice steer prices on Mondays in cents per hundredweight. In each equation, the first difference of the Omaha price is the explanatory variable.
[b] Standard errors of coefficients in parentheses.
[c] Denver observations available only for thirty-eight continuous weeks. Sioux City equation uses same time period.

[5] Taken as a whole, all of the results indicate unusual price behavior in 1959 and 1960. In Sioux City-Omaha equations using price levels, the intercepts are two standard deviations above zero. In the first difference equations, the slopes differ from one. For the Denver-Omaha equations, the slopes are consistently below one. Since years before and after have the expected relationships, the performance in 1959 and 1960 is a puzzle. The answer to the puzzle is not within the scope of this paper.

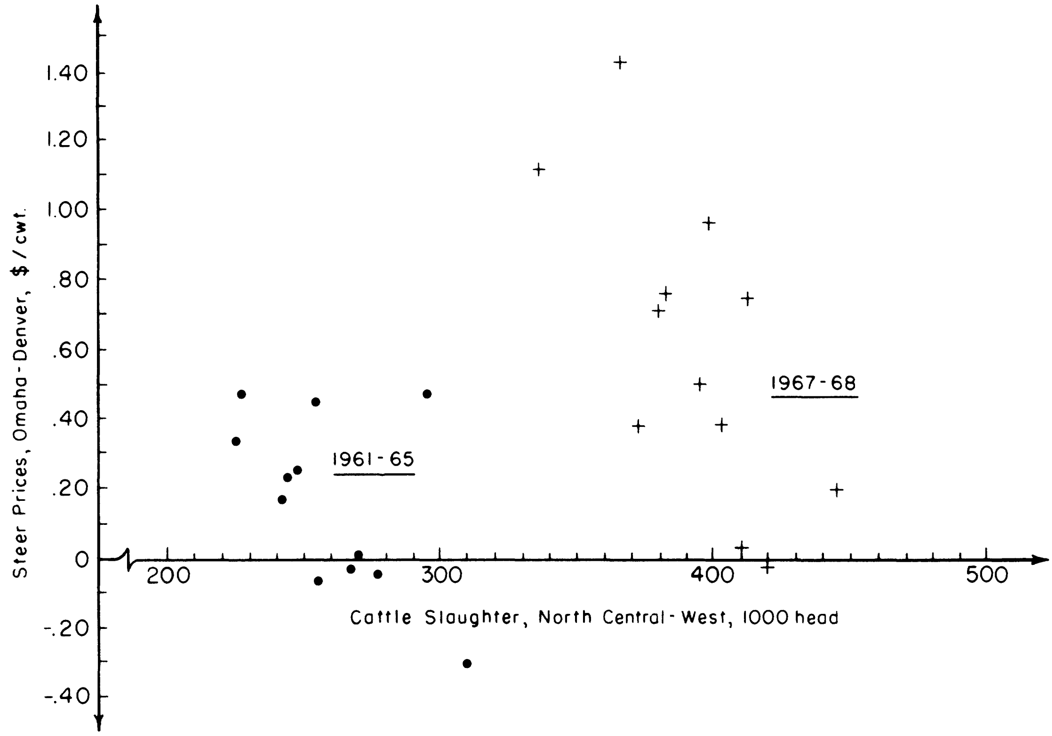

Figure 1. Regional differences in steer prices versus regional differences in cattle slaughter

exhaustive investigation of this hypothesis would require construction of a spatial equilibrium model, an analysis beyond the scope of this paper. Moreover, the regional supply variables used here are crude, and errors in measuring these variables may account for the shift.

Concluding Observations and Qualifications

This paper defines thinness by analogy with sampling concepts from statistics. This approach establishes a (perhaps arbitrary) standard of precision (price behavior), and then estimates the number of transactions required to obtain the standard. The approach has the advantage of defining what is meant by "precise" price behavior and of emphasizing the relative nature of defining a thin market. The structure of prices is exceedingly complex, however, and it may be difficult to estimate the number of transactions, n, required to avoid thinness. (An alternative application is to estimate the precision implied by existing volume, n.)

Given the complexity of establishing prices for live cattle, the number of transactions required to discover steer prices with considerable precision seems small to me. Apparently thirty to thirty-five transactions per Monday (about 50,000 steers per year) were adequate under the conditions of the 1960s to discover Denver prices rather precisely relative to the Omaha price. If a specific assumption were made about the distribution of price changes, the number would be even smaller.

The reader, however, should not be overly reassured by these results. Since the 1960s, commodity prices have become more variable. If a fixed standard of precision is maintained, such as 10¢ per hundredweight, then the number of public transactions must increase with the variance. In contrast, the trend has been toward declining volumes on central cash markets.

Estimates of the variance of changes in daily or weekly price from a sample of fifty-two weeks may be inappropriate for estimating n. The variance presumably should reflect changes in the current forces determining prices. If there is little reason for prices to change, then few transactions are needed to achieve a given level of pricing precision (and vice-versa). Thus, a sample based on a calendar year may not be from a population relevant for estimating the variance needed for obtaining n. Intraday prices or a shorter period may be more appropriate (but see note 4). Also, prices varied little in the 1955–68 period relative to the 1970s, and even for the period studied, the variances of weekly prices by years were unstable. The question of an appropriate sample period for estimating a relevant σ^2 is not fully answered in this paper.

In interpreting the results, the distinction between a market and a market place or a market channel must be kept in mind. Denver was just one market place in a national cattle market. Thinness in Denver does not, of course, necessarily imply thinness in the market in general. But, to the extent publicly reported prices are needed to establish regional or quality differentials, a problem of thin markets may remain. The Denver market place became a poor place to discover regional prices for choice steers relative to the Omaha market place. Because, during the period studied, fed cattle production was growing in Colorado, regional price differences were probably better established in a market channel other than the Denver terminal market, but it may be more costly to obtain public price reports from channels other than central markets. If the principal role of publicly reported prices is to establish a known, national base for other transactions, then the increased volume of trading on futures markets may assist in establishing an accurate base (Cox).

Another issue is whether the items priced on the central market are representative of the full population of items being sold. It is alleged, for example, that the lots sold on central markets tend to be lower quality than the typical lots of the product. The observed prices may not be a random sample of the population prices. For pricing, however, a random sample may be unnecessary. If the items priced on a central market are of a known quality, then the price established for that quality can be used to price other quality products. The remaining prices need only establish the correct difference. Thus, bias is permitted if a correction factor can be obtained, but the assumption $E(X) = \mu$ rules out the use of manipulated prices in estimating the true equilibrium.

With respect to performance of prices on a terminal market, thinness can mean that no transactions occur over extended periods of time. Consequently, the descriptive statistics, such as the mean and variance, become more difficult to interpret. For the period with continuous price series, r^2 between Omaha and Denver prices declined, and the slope coefficient moved further from one. From a technical statistical viewpoint, this does not necessarily mean prices are biased; rather one may say that prices are established with less precision as volume declines. Individual prices and their averages have larger errors, but these errors can be offsetting so that, on average, there is no bias.

In evaluating the results, it is not appropriate to talk in terms of formal hypothesis tests. A given data set was subjected to many exploratory analyses, thereby reducing levels of significance (Wallace). Nonetheless, the results are clearly compatible with the hypothesis that declining volume results in imprecise pricing. This is the implication of the analysis of Denver prices, strengthened by the Sioux City results. Moreover, logic alone suggests that declining information will result in larger pricing errors.

Given the historical nature of the analysis, the empirical results are not immediately applicable to a current problem, but the results do seem to point toward useful methods of studying thin markets.[6] It may be possible to estimate the number of transactions required for a given level of pricing precision or to estimate the precision implied by the existing volume. Such estimates can be sharpened over those of this paper, if convincing evidence is provided that price changes have a specific probability distribution and if an appropriate sample can be ascertained for estimating σ^2. Other research needs to consider the costs of improving pricing precision and to suggest less arbitrary ways of defining the precision parameter, c. For example, the concept of price risk might enter into defining c. Many of the problems of analysis found in this paper seem surmountable, and analyses of current issues, such as the pricing of beef carcasses, could be attempted.

[Received May 1979; revision accepted January 1980.]

[6] Several reviewers have suggested that the sampling analogy gives a misleading definition of a thin market. For example, the mere use of number of transactions misses the possibility that one transaction may contain more information than another; all transactions are not equal in the amount or quality of information provided. But such an objection might be overcome by extending the results of this article to include stratified sampling, or if nothing else, particular markets might be analyzed in terms of their ability to provide judgment samples. In the author's view, it is important to develop a concept of a thin market that has empirical content. The sampling framework is one approach to empirical analysis, and hopefully readers will be motivated to make extensions and improvements.

References

Cox, Charles C. "Futures Trading and Marketing Information." *J. Polit. Econ.* 84(1976):1215–37.

Cramér, Harald. *Mathematical Methods of Statistics.* Princeton, N.J.: Princeton University Press, 1946.

Hayenga, Marvin L., ed. *Pricing Problems in Food Industry (with Emphasis on Thin Markets).* North Central Project 117, Monograph 7, 1979.

Hess, Alan C. "Experimental Evidence on Price Formation in Competitive Markets." *J. Polit. Econ.* 80(1972):375–85.

Mann, Jitendar S., and Richard G. Heifner. *The Distribution of Short-Run Commodity Price Movements.* USDA Tech. Bull. 1536, Mar. 1976.

Mood, A. M. *Introduction to the Theory of Statistics.* New York: McGraw-Hill Book Co., 1950.

Paul, Allen B. "Some Basic Problems of Research into Competition in Agricultural Markets." *Amer. J. Agr. Econ.* 61(1979):170–7.

Samuelson, Paul A. "Proof that Properly Anticipated Prices Fluctuate Randomly." *Ind. Manage. Rev.* 6(1965):41–9.

Smith, Vernon L. "An Experimental Study of Competitive Market Behavior." *J. Polit. Econ.* 70(1962):111–37.

Trierweiler, John E., and James B. Hassler. "Measuring Efficiency in the Beef-Pork Sector by Price Analysis." *Agr. Econ. Res.* 23(1971):11–17.

Wallace, T. Dudley. "Pretest Estimation in Regression: A Survey." *Amer. J. Agr. Econ.* 59(1977):431–43.